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Gait Optimization of Biped Robot during Double Support Phase by Pure Dynamic Synthesis

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Abstract: This paper deals with dynamic optimization of biped locomotion. The main focus of this research is motion optimization of double support phase. The optimization problem is dealt by using Pontryagins Maximum Principal. For motion optimization of double support phase, the closed kinematic chain has been considered to be opened at appropriate joint and the components of ground reaction forces has been applied on the tip of front leg and finally the penalty method has been used to tighten the leg to its prescribed location. The feasible sets of motion are taken into consideration by using inequality constraint to limit the joint motion. Also the components of ground reaction forces on front leg have been introduced as control variables in optimization of double support phase. The proposed technique has the ability to generate optimal free motions without specifying joint trajectories and minimized the performance criterion based on joint actuating torques. The two point boundary value problem has been solved by implementing a shooting method. This technique allows for specifying a few parameters to characterize gait pattern. The optimization process has the ability to generate a motion with a minimum of postural and kinematics data. Unlike previous research which used computational intelligent techniques for biped gait optimization, this study focuses on development of purely dynamic synthesis of biped motion during the double support phase.

Keywords: Pontryagins Maximum Principal, Dynamics of Walking, Optimization.

INTRODUCTION

In recent years, have been considerable attentions to study biped robot. Particularly have been increasing enthusiasms to research about the lagged locomotion in both areas of robotics and biomechanics [1-8]. For this reason, it is promising the use of biped robots in human environments as well as the development of biped robot's control algorithms [4, 6, 9-18]. For this reason there are two approaches: one is using computational intelligent techniques for control and optimization [13, 14] and the other is improving classical dynamic solutions for mastering the dynamics of a multi-body system with sophisticated kinematics [4, 6, 9-18]. This research has been focused on the second approach.

From the review of previous literature, it is believed that the biped with the simplest kinematics was designed by McGeer as a compass link structure which is able to perform a purely sagittal gait. That biped robot walk down a slope by gravity induced passive motion [19]. Thereafter the dynamics of five link sagittal biped has been modeled for designing impulsive control in double support phase as a result the energy expenditure has been decreased; although the impact effect has not been considered due to impulsive motion control [20]. Another technique for gait optimization of biped robot has been developed, on the basis of representation of join trajectories by polynomials which coefficients are adjusted for minimizing energy expenditure [21]. Further research has been carried by defining set of pattern parameters that included the specification of kinematic transfer conditions through trajectory synthesis during single support phase of a seven link anthropomorphic robot [22]. Some optimization techniques are similar approach based on kinematic specifications [22- 24].

A piecewise constant inputs method has been implemented for energy optimization during gait cycle[25, 26].

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This paper focuses on achieving a pure dynamic synthesis of biped robot gait during both single support phase [21- 23] and double support phase [25] on sagittal plane without considering the impact effect at the end of the swing phase [2, 24]. This approach allows for a frilly dynamic model of the biped which is based on minimizing the integral of quadric joint actuating torques. Gait optimization synthesis is achieved by applying the Pontryagins Maximum Principal. The single support phase can be modeled as an open kinematic chain although if the tips of this open kinematic chain is brought into contact with floor, a closed-loop will be created as a multi support phase, consequently the differential equations will become more complicated.

For dynamic simulation of multi-body system, the closed-loop can be considered as open at constrained joints. These conditions can be used for formulating dynamic model with Lagrangian multiplies. As a result, a set of differential equations will be obtained which are not suitable for dealing with dynamic optimization. The proposed approach is based on applying penalty method which releases optimization problem from Lagrangian multipliers. In this method, all closed loops of the multi-body system have been considered as open at appropriate joints. The constraints which express the closure condition have been used in the performance criterion of the optimization problem in order to minimize the optimization criteria; thereby the numerical value of the mentioned constraints will be reduced. At the end of this paper, typical result of this method of a gait cycle has been simulated.

MATERIALS AND METHODS

Kinematic Model: A sagittal model of a five degree of freedom, anthropomorphic biped has been shown in Fig. 1. This model is contained of five links; they are numbered from L_1 to L_5. The mass of each link is defined by m_i and I_i^z represents the moment of inertia with respect to the joint axis O_i. Such a planar system comprises of two ankles, two knees and two coaxial hip joints. The biped motion has been considered by the five relative joint coordinates which have been adapted to generalized coordinates. The joint coordinates and joint velocities have been noted as:

$$q = [q_1, \dots, q_n]$$
$$\dot{q} = [\dot{q}_1, \dots, \dot{q}_n]$$

In this model n is 5.

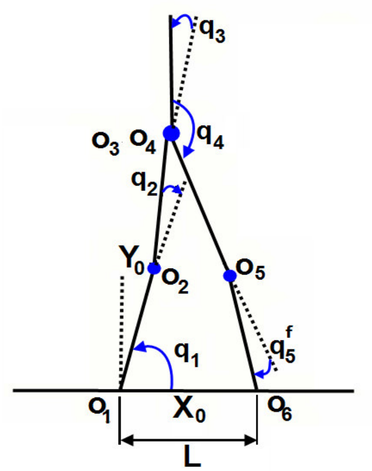

Fig. 1: Biped kinematic model.

Dynamic Model: For formulating the biped dynamics in the double support phase, the set of constrains have been assumed to be holonomic. The biped leg must be fixed at its prescribed location during the double support phase. The geometrical constrains have been written in equation (1) with respect to generalized coordinates q_i.

$$\Phi(q)=\begin{pmatrix}\Phi_1(q)\\ \Phi_2(q)\end{pmatrix}=0 \quad , \qquad \Phi(q)\in\Re^m, m=2 \qquad (1)$$

By using Lagrange formulation, dynamic equation of motion in accordance to applying Lagranigain multipliers has been written in equation (2).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=Q_i^a+Q_i^d+J_q^T\lambda \ , \qquad i=1,\dots,n \qquad (2)$$

As it is illustrated in Fig. 1, Q_i^a (resp. Q_i^d) represents the joint actuating torques (resp. joint dissipative torque) exerted by L_{i-1} on L_i at O_i , J_q^T represents the Jacobean matrix and λ represents the forces of constraint which are vertical and horizontal ground reaction forces. Since Pontryagins Maximum Principal has been implemented for motion optimization, the main problem in formulating dynamic model of double support phase in comparison with single support phase is that, we are consorting the Lagrangian multipliers which are not suitable to be used in Pontryagins Maximum Principal. As illustrated in Fig. 2, the approach to overcome this difficulty is to consider closed kinematic chain of the biped to be opened at joint O_6 and applying the components of ground reaction forces on the tip of front leg and finally using the penalty method to tighten the tip of the leg on its prescribed location. In this method the components of reaction forces would be considered as control variables same as actuating torques. Using penalty techniques holds the fact that firstly the Lagrangian multipliers should be replaced by reaction forces in equation (2) and secondly motion optimization should be done with respect to geometrical constraints which minimizes $\phi(q)$.

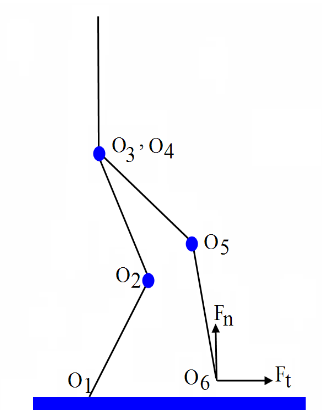

Fig. 2: Considering closed kinematic chain of biped to be opened in double support phase.

By these assumptions the equation (2) can be written as equation (3).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=Q_i^a+Q_i^d+J_q^T F \qquad (3)$$

Where F stands for the ground reaction forces. There is a need to underline that it is computationally quite efficient to formulate a dynamic model adapted at the best to the selected optimization technique. Since Pontryagins Maximum Principal has been used for solving the dynamic optimization problem thereby the implementation of the Pontryagins Maximum Principal requires the formulation of the dynamic model in the state space form. The Hamiltonian dynamic model is suitable for fulfilling the requirement and more importantly it strengths the robustness of the optimization algorithms [2]. The outline of the required formulation for defining the conjugate momentum and Hamiltonian has been mentioned in equation (4).

$$P_i=\frac{\partial L}{\partial \dot{q}_i} \qquad i=1,\dots,n$$
$$H(q,p)=p^T\dot{q}-L(q,\dot{q}) \qquad (4)$$

Lagrange's equations can be formulated in Hamiltonian form which has been mentioned in equation (5).

$$\dot{q}_i=\frac{\partial H}{\partial p_i}$$
$$\dot{p}_i=-\frac{\partial H}{\partial q_i}+Q_i^a+Q_i^d+J_q^T F \qquad (5)$$

The expression of p can be written through equation (4) as $p=A\dot{q}$ in which A is the $(n\times n)$ mass matrix of kinematic chain. Then equation (3) becomes more explicit.

$$\dot{q}_i=\sum_{j=1}^{n}A_{ij}^{-1}p_i$$
$$\dot{p}_i=-\frac{1}{2}p^T A_{,i}^{-1}p-V_{,i}+Q_i^a+Q_i^d+J_q^T F \qquad (6)$$

Where V stands for the gravity potential and:

$$A_{,i}^{-1}\equiv\partial A^{-1}/\partial q_i \qquad V_{,i}\equiv\partial V/\partial q_{,i} \qquad (7)$$

With this formulation, Hamiltonian equations are perfectly structured for applying the Pontryagins Maximum Principle. The state and control variables have been defined in equations (8, 9).

$$X=(x_1,\dots,x_{2n})^T\equiv(q_1,\dots,q_n,p_1,\dots,p_n)^T \qquad (8)$$

$$u=(u_1,\dots,u_{n+m})^T\equiv(Q_1^a,\dots,Q_n^a,F_n,F_t)^T \qquad (9)$$

Where u_1 to u_n represent joint actuating torques, F_n represents the normal and F_t represents the horizontal component of ground reaction force in Fig. 2. The double set of vectorial equation (6) can be reacts as the second order differential vector equation.

$$\dot{x}(t)=F(x(t))+B(x(t))u(t) \qquad (10)$$

In equation (10) initial and final states have been specified as equation (11).

$$x(t^i)=x^i, \qquad x(t^f)=x^f \qquad (11)$$

Feasible Motions and Constraints: Feasible motions of the biped are defined by two types of specific conditions. The first type consists of limiting the joint actuating torques. The second type specifies interaction conditions between the stances foot and the ground. Torqueses produced by actuators have limited values. They are considered at the joint level as equation (12).

$$\forall t\in[t^i,t^f], \qquad \left|Q_i^a(t)\right|\le Q_i^{a,max} \qquad (12)$$

The vertical component of ground reaction forces must remain positive during the motion due to unilaterality of contact. This condition means that the foot is not stuck on the ground and the ground can only push it. Therefore the unilaterality condition is expressed by equation (13).

$$\forall t\in[t^i,t^f], \qquad 0<F_n^{min}\le F_n(t) \qquad (13)$$

The latter condition can be expressed by ignoring the slide of the foot on the ground in equation (14).

$$\forall t \in [t^i, t^f], \qquad |F_t(t)| \le \mu\, F_t(t) \qquad (14)$$

Equations (12), (13) and (14) define the space of control variables U.

Formulating an Optimal Control Problem: An optimal motion can be generated in a gait cycle by minimizing a performance criterion that represents dynamic cost. In optimization, there are two alternatives: minimizing actuating torques or energy expenditure. Since the biped stands and moves in a vertical plane, it is submitted to the gravity. For this reason, the first alternative has been supported by introducing the integral cost in equation (15).

$$J(u) = \int_{t^i}^{t^f} L(x(t), u(t))dt \qquad (15)$$

Where the Lagrangian is the quadric function of the normalized control variables u_i in equation (16).

$$L(x,u) = \frac{1}{2}\sum_{i=1}^{n+m} \xi_i (u_i / u_i^{ref})^2 \qquad (16)$$

Where ξ_i are weighting factors and u_i/u_i^{ref} represent dimensional joint actuating torques and dimensional ground reaction force. The reference value of the reaction forces has been assumed as biped's weight. The weighting coefficients play an important role. By increasing ξ_i, the optimal corresponding u_i are decreased. Thereby there is possibility to reduce the action of actuating torques and master the ground reaction forces which are applied on the tip of front leg.

Dealing with the Geometrical Constraints: The geometrical constraints which have been defined in equation (1) can be dealt by using computational techniques such as penalty method which has been developed through mathematical programming. The penalty method can minimize the geometrical constraints functions. In this method the geometrical constraints must be added to optimization criteria as a quadric term which has been defined in equation (17).

$$J_r(u) = J(u) + \frac{r}{2}\int_{t^i}^{t^f} \| \phi(x) \|^2 \, dt \qquad r > 0 \qquad (17)$$

The function J_r must be minimized by sufficiently great value of the penalty multiplier r.

Applying Pontryagin's Maximum Principle: The minimization problem can be summarized as: finding a phase trajectory $t \rightarrow x(t)$ and a control vector $t \rightarrow u(t)$ for minimizing J_r, which has been specified in equation (18) and satisfying the equation (10) in consideration of equation (11).

$$u \in U \qquad \underset{r \ great}{Min} \ J_r(u) \qquad (18)$$

The Pontryagins function can be defined as equation (19)

$$w \in R^{2n} \qquad H(x,u,w) = w^T(F(x) + B(x)u) - L_r(x,u) \qquad (19)$$

The maximum principle [28] states that if $t \rightarrow (x(t), u(t))$ is a solution of equations (18, 19) then there is a costae function $t \rightarrow w(t)$, $w \in R^{2n}$ that satisfies the costae equation (20) and maximal condition has been defined in equation (21).

$$\dot{w}(t)^T = -\partial H / \partial x \qquad (20)$$

$$v \in U \qquad H(x,u,w) = max\ H(x,v,w) \qquad (21)$$

A prominent benefit of the Pontryagins Maximum Principal lies in equation (21) which allows the constraint on $u(t)$ to be completely satisfied and yields an explicit expression of the optimal control variables through equations (10, 16, 19) [2]. The unknown functions x and w appear as a solution of a 4n-order differential system of the type in equation (22) accompanied by the boundary conditions mentioned in equation (11).

$$\forall t \in [t^i, t^f], \qquad \begin{aligned} \dot{x}(t) &= F_1(x(t), w(t)) \\ \dot{w}(t) &= F_2(x(t), w(t)) \end{aligned} \qquad (22)$$

Here, there is a problem of a two-point boundary value.

RESULTS AND DISCUSSION

Numerical Simulations: The two point boundary value problem can be solved by computational techniques such as finite difference algorithms or shooting methods. The latter approach has been selected for its efficiency and simplicity of implementation that so-called transition matrix method [27]. Due to the intensive non-linearity of dynamic equations, the main difficulty is to overcome the algorithms in order to converge toward an optimal solution, which consist a sufficiently accurate guess. The first-order gradient algorithms have been used to overcome this difficulty [27]. According to Fig. 1 the biped model specification has been defined in Table 1.

Table 1: Dimensional characteristics of the biped.

Link	1	2	3	4	5
Mass(kg)	6.4	8.6	55.0	8.6	6.4
Length(m)	0.4	0.41	0.55	0.41	0.4
Center of Gravity	0.24	0.27	0.33	0.14	0.16
I_i^z	0.44	0.69	7.0	0.23	0.25

The presented techniques in this study can be used to simulate single support and double support phase. The main difference of double support phase in comparison with single support phase is that the closed kinematic chain has been considered to be opened and applied the components of ground reaction forces on tip of the leg. Consequently in double support phase, the penalty factor has a great numerical value but in single support phase this factor is zero. The simulated optimal motion has been shown in Fig. 3. The step length is equal to 0.40 (*m*) and total motion time is 0.43 (*s*). The average horizontal hip velocity in single support phase is equal to 0.95 (*m/s*) and in double support phase is equal to 1.0 (*m/s*), which is equal to average speed of human gait. The impact phase at the end of single support phase has been ignored [2].

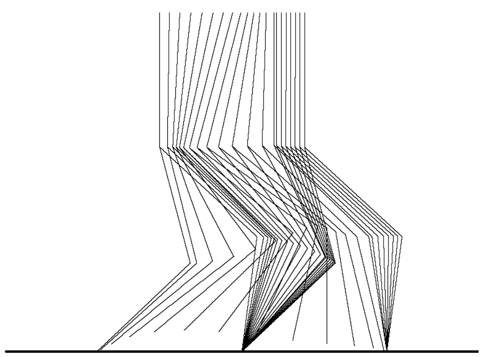

Fig. 3: Optimal motion of the biped during a complete gait cycle for step length of 0.4 (*m*).

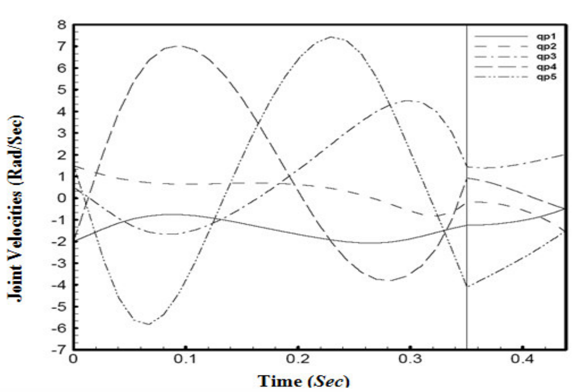

Fig. 4: Joint relative velocities.

The variation of joint relative velocities and actuating torques has been shown in Fig. 4 and 5. The ankle actuating torque of stance leg has been saturated at the end of double support phase. Introducing a sufficiently great value of the weighting factor ξ_i in equation (16) weaken the ankle torque of stance leg during double support phase. Fig. 6 shows the components of ground reaction forces on both legs during both phases. In double support phase the piped weight has been slightly transferred to front leg. Since the components of reaction forces on front leg in double support phase have been considered as control variables, therefore this method allows mastering theses components directly.

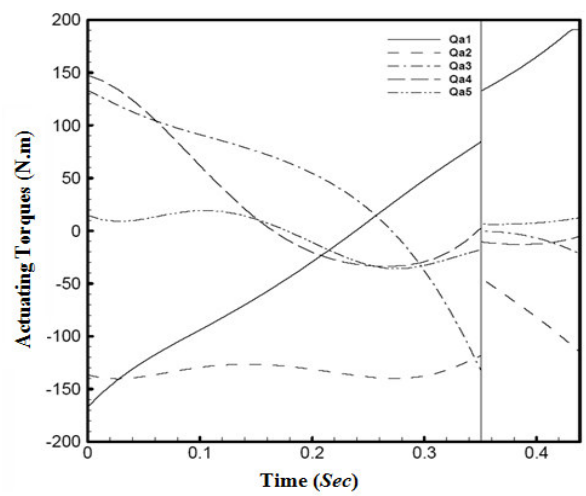

Fig. 5: Time variation of actuating torques.

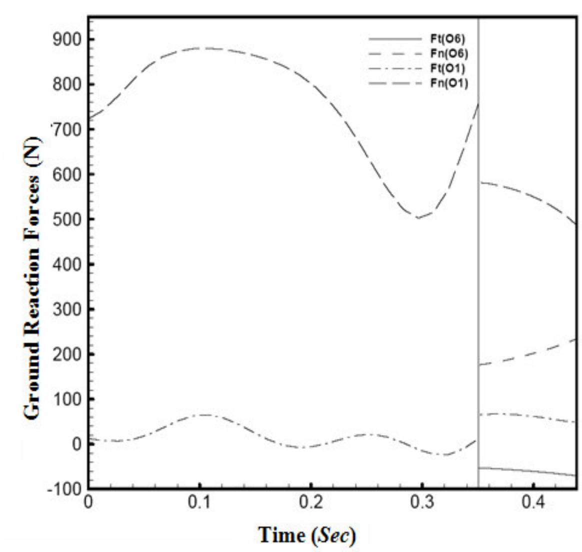

Fig. 6: Time variation of ground reaction forces.

Equation (16) has been used for computing energy consumption.

$$E = \int_{t_i}^{t_f} \sum_{i=1}^{n} \left| \dot{q}_i(t) Q_i^a(t) \right| dt \qquad (23)$$

The energy expenditure during single support phase is 200.2 (J) and during the double support phase is 23.5 (J).

CONCLUSION

The optimal motion of a biped robot during a complete gait cycle has been presented. The Pontryagins Maximum Principal has been implemented for motion optimization of both single support phase and double support phase. The closed kinematic chain in double support phase has been considered to be opened and geometrical constraints has been dealt by means of penalty technique. The presented technique allows generating smooth motions with minimum kinematical constraints and mastering the control variables directly as interaction forces. After optimization, the step length is 0.40 (m) and total motion time is 0.43 (s). The average horizontal hip velocity in single support phase is equal to 0.95 (m/s) and in double support phase is equal to 1.0 (m/s), which is equal to average speed of human gait; therefore the accuracy of our method has been confirmed.

The focus of the future work would be on the development of computational techniques for gait optimization in three dimensions.

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