



## Physical Characterizations of Sands and Their Influence in Fall Velocity

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**Abstract:** Cancun beach is one of the most important tourist resorts on the Mexican coast, and is of prime importance to the local and national economies. As a result of the intense use of the beach and the unavailability of sand reserves, the extreme weather events that occur in the region (hurricanes) have permanently damaged the beach. Over the last two decades several strong hurricanes severely weakened the system, but hurricane Wilma, October 2005, caused devastating erosion to the beach. After Wilma, an emergency nourishment project was implemented pumping a volume of 2.7 million m<sup>3</sup> of sand onto the beach. Clearly, the sand used for the nourishment has a different shape factor, diameter distribution and density property compared to the native sand. The main goal of this paper is to characterize and compare the fall velocities of sand samples taken in and around Cancun. The settling velocities obtained for several samples were compared with the empirical formulations proposed by various authors. The influence of several parameters measured in the laboratory is discussed and a new formulation is presented.

**Keywords:** Settling velocity, Cancun sands, sediment physical properties

### INTRODUCTION

Cancun is a 17 km long, barrier island located in the North East of the Yucatan peninsula, in the state of Quintana Roo, on the Mexican Caribbean (Fig. 1). The development of this resort over the last 40 years has broken several natural equilibriums. As a result, the effects of a series of strong hurricanes produced chronic erosion problems between Punta Cancun and Punta Nizuc. Following the devastation of hurricane Wilma (2005), a beach nourishment program was begun in 2006 with 2.7 million cubic meters of sand used to nourish the beach system. The material employed to fill the beach has different mechanical properties compared to the original sand and that found in the surrounding area. It is now very important to estimate the potential behavior of this new beach.

The settling velocity of beach material is a very important parameter when analyzing the stability of a beach; the processes of advance or retreat of a coastline under a given wave climate are obviously influenced by it. Traditional theoretical formulae for estimating fall velocity considered sand grains as spherical<sup>[1]</sup>. This has since been demonstrated to be a false assumption which over-estimates its relation to settling velocity. Overcoming the spherical assumption, many other formulae have been developed, i.e. those by Graf<sup>[2]</sup>,

Hallermeier<sup>[3]</sup>, Dietrich<sup>[4]</sup>, van Rijn<sup>[5]</sup>, Soulsby<sup>[6]</sup> and Ahrens<sup>[7]</sup>; all of which were derived empirically but validated with the same data set used to get the equations. The common path of the aforementioned formulae depends on sediment diameter and shape factor (some of them with the usage of the nominal diameter). But a simple look at the distribution of the physical properties of the sand along the beach and across the beach profile, in combination with the wave energy, indicates that the fall velocity may also be governed by other parameters.

This paper presents the results of field and laboratory work conducted to investigate the influence of physical properties of sand samples taken from Quintana Roo, Mexico and their relation to fall velocity. The analysis of all the data compiled gives an idea of the parameters involved in the settling process and the physical characterization of the sand studied.

### FIELD WORK

The sand transport along and across a beach is difficult to understand because of the variety of processes involved and the ways in which these occur. Nevertheless, once the material is deposited at the bottom, the distribution of the sand properties and grain sizes can be used to get an idea of the equilibrium state

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of the beach. For example, in a stable beach, the bigger particles should be found above the mean sea level, while in the surf zone more rounded grains are expected to appear. In turn, an eroding beach may have particles with a higher flotation index close to the swash zone.

In order to get a comprehensive view of the grain size and physical properties distributed across various beach profiles and along a 40km stretch of coast, several sand samples were taken in Quintana Roo, Mexico from Punta Sam to Puerto Morelos (see Fig. 1).

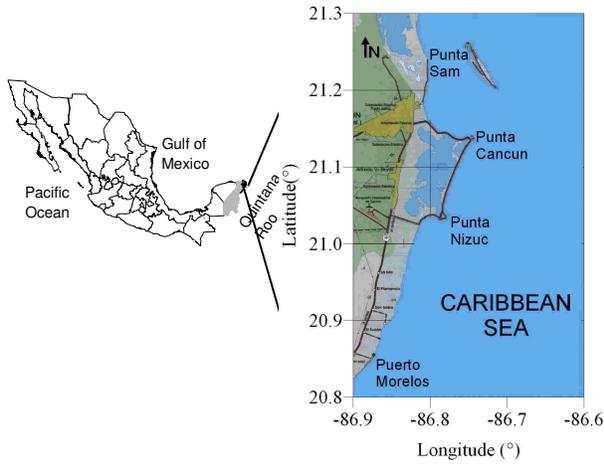


Fig 1: Location of the littoral cell of interest

In planning the sampling logistics and procedures, the main idea was that at least three points across the beach profile should be sampled 1) at the beach berm, 2) at the swash zone and 3) close to the breaking zone (approx. 1 m below the mean water level). Each of these samples afford specific information, that is, the berm shows the material that has not been moved by the waves or tides and is expected to have been settled for a longer time than in the other zones. The sample from the swash zone gives information related to the mixture of the sand and the water, this material has been carried by the breaking waves during its energy loss process. Finally the breaking zone supplies data on the material that has stayed at the bottom after the steeping and breaking of the waves. All the samples were taken at the sand surface and the approximate weight of each sample was 2.0 kg.

The sampling sites along the coast line were selected, assuring that at least one sample was taken from every littoral cell between Punta Sam and Punta Nizuc. The three samples at Puerto Morelos were taken

as representative of the littoral cell there. Each site was identified with the UTM coordinates given by a manual DGPS so that the same places can be sampled again in the future. In Table 1 the ID codes and coordinates of the 22 sample points are listed.

The ID number is completed with a letter B, L or S which stands for berm, swash and breaking zone, respectively.

Figures 2 to 4 show the position of each sampling site between Punta Sam and Punta Cancun (A to G), between Punta Cancun and Punta Nizuc (Boca Sur and P1 to P10) and at Puerto Morelos (PM1, PM2 and PM3), respectively.

Table 1: Sampling points list and UTM coordinates

Number	ID	UTM	
		x	y
1	Boca Sur	521,259.9992	2,326,122.1175
2	PM1	513,801.1374	2,307,548.0139
3	PM2	513,943.9119	2,307,851.8631
4	PM3	513,664.9397	2,307,322.5620
5	1	526,453.7885	2,336,940.7871
6	2	526,150.4047	2,336,595.8547
7	3	526,017.6126	2,336,409.6325
8	4	525,154.7245	2,334,853.4970
9	4A	524,605.0000	2,333,629.0000
10	5	524,218.9638	2,332,622.0587
11	6	523,456.9477	2,330,419.6613
12	7	522,951.5196	2,328,850.9625
13	8	522,652.5128	2,327,388.3438
14	9	522,664.9844	2,326,847.3934
15	10	522,860.8821	2,326,243.7693
16	A	525,843.9993	2,337,619.1441
17	B	526,088.0000	2,337,574.0283
18	C	523,442.0098	2,338,090.0004
19	D	522,114.0006	2,338,094.0011
20	E	522,285.0002	2,338,171.9409
21	F	520,133.9994	2,341,716.0157
22	G	520,531.1592	2,348,845.8938

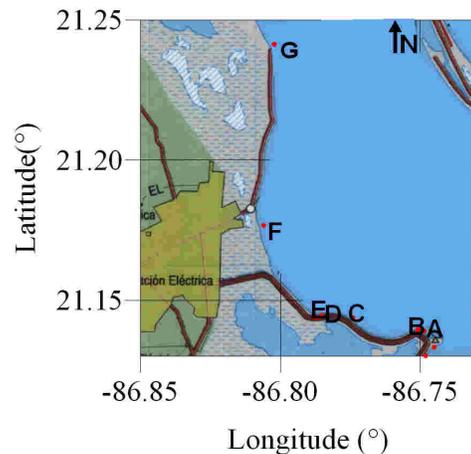


Fig. 2: Sampling points between Punta Sam and Punta Cancun

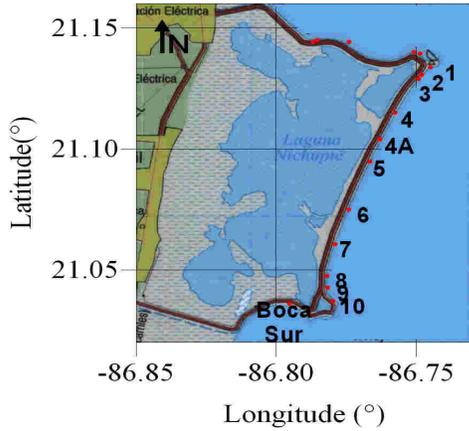


Fig. 3: Sampling points between Punta Cancun and Punta Nizuc

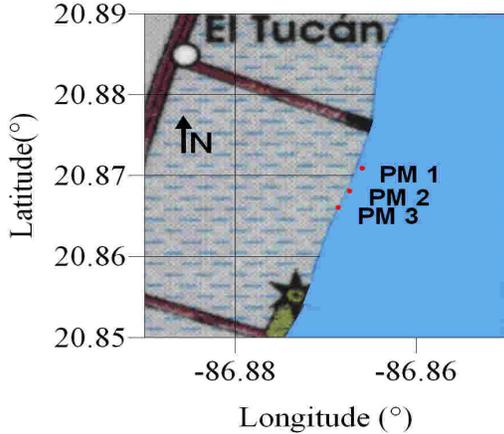


Fig. 4: Sampling points at Puerto Morelos

A total of 60 sand samples were characterized in the Coastal and Port Laboratory of the Institute of Engineering at the National Autonomous University of Mexico.

### LABORATORY WORK

Most of the sand samples arrived at the laboratory damp or wet. To dry out the samples two processes were used: 1) a pre-drying, which consisted of heating the samples using very intense light for 12 to 24 hours and 2) full drying in an oven at 110°C for 10 to 12 hours. Once the samples were completely dried, the net weight could be measured. The samples were then analysed, characterizing grain size distribution, density, Corey shape factor and settling velocity.

The grain distribution characterization was carried out using a series of 15 sieves (16, 4.00, 2.36, 2.00, 1.40, 1.00, 0.71, 0.50, 0.355, 0.250, 0.180, 0.125, 0.090, 0.063 and 0.053 mm) which were shaken with an electric Ro-Top machine for 7.5 minutes. The material

retained in each sieve was weighed with an electronic scale. Examples of some of the curves obtained can be seen in Figs. 5 and 6.

All the samples contained at least 90% sand. In most cases the additional material was very fine, although in some samples big particles of organic material were also present.

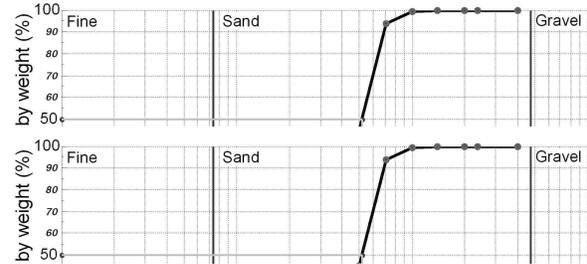


Fig. 5: Grain distribution curve for sample PL 1

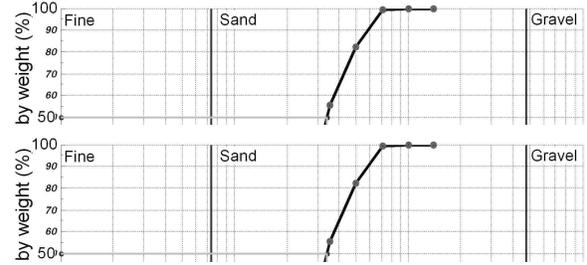


Fig. 6: Grain distribution curve for sample PB 1

To find the density of each sand sample the sand retained by sieve No.5 (4mm) was used and divided into two samples,  $S_1 = 0.04$  kg and  $S_2 = 0.02$  kg. Two empty 100 and 50 ml volumetric flasks were weighed,  $W_{s1}$  and  $W_{s2}$ , respectively, while  $W_{fw1}$  and  $W_{fw2}$  correspond to the weights of the flasks when full of distilled water. Sands  $S_1$  and  $S_2$  were poured into the flasks and the excess water removed. Both flasks were then put into boiling water for 15 to 20 minutes, and then taken out. Once the flasks were at room temperature, the volume of the mixture was modified with distilled water to 100 or 50 ml and the flasks were again weighed,  $W_{fsw1}$  and  $W_{fsw2}$ . The relative density of the sand was thus estimated:

$$S_s = W_s / (W_s + W_{fw} - W_{fsw}) \quad (1)$$

The average of  $S_{s1}$  and  $S_{s2}$  was taken as the density of each sample. Table 2 shows some of the densities obtained. (PL = breaking zone; PS = swash zone; PB = beach berm)

The shape factor was measured using digital photographs of a small amount of sand with a grain size close to  $D_{50}$  (the material retained by the first sieve is finer than the  $D_{50}$ ). The photographs were analyzed one by one and the dimensions length,  $a$ , width,  $b$  and thickness,  $c$ , of the grains obtained with a MATLAB

process. The Corey shape factor, *CSF*, was then computed as:

$$CSF=c/(ab)^{1/2} \quad (2)$$

Table 2: Densities of selected sand samples

ID	$S_s$	
PL PM1		2.668
PS PM2		2.721
PB 1	2.759	
PB 4A		2.658
PL 5	2.817	
PS 7	2.649	
PL 9	2.768	
PB 10		2.540
PS C	2.749	
PB F	2.749	
PR F	2.417	
PL F	2.210	
PS F	2.532	

Figure 7 shows an example of photographs taken with a micro lens, and Table 3 lists some Corey factors.

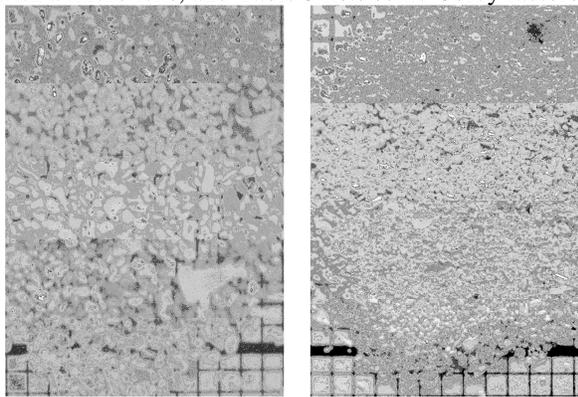


Fig.7: Examples of sand grain photographs. Sample PL 5 (left) and sample PR PM2 (right). Grid mesh = 1 mm.

Table 3: *CSF* of selected sand samples

ID	<i>CSF</i>	
PL PM1		0.6815
PS PM2		0.7545
PB 1	0.7452	
PB 4A		0.7600
PL 5	0.6158	
PS 7	0.6974	
PL 9	0.7869	
PB 10		0.6270
PS C	0.6847	
PB F	0.7344	
PR F	0.6083	
PL F	0.8118	
PS F	0.6558	

The settling velocity was measured manually using a static water recipient. Sand grains with sizes close to  $D_{50}$  were left on the water surface and then touched lightly, just enough to break the surface tension and allow them to drop down and the time was recorder in milliseconds and the distance in millimeters. Some results are shown in Table 4.

## DISCUSSION

In order to characterize settling velocities a comparison was made between the measured settling velocities and the values given by other authors. The formulations by Rubey<sup>[8]</sup>, Hallermeier<sup>[4]</sup>, van Rijn<sup>[5]</sup>, Julien<sup>[9]</sup>, Cheng<sup>[10]</sup>, Soulsby<sup>[6]</sup>, Ahrens<sup>[7]</sup>, Chang & Liou<sup>[11]</sup> and Jimenez & Madsen<sup>[12]</sup> were used. Figure 8 shows the match of the measured data with the nine empirical formulae.

At first sight the values given by van Rijn<sup>[5]</sup>, Julien<sup>[9]</sup> and Cheng<sup>[10]</sup> are very close to the measured velocities. In order to achieve a quantitative dimension, the average relative error of each formula was calculated. These errors are presented in Table 5.

It is clear from Table 5 that the formula that best fits with the measured data is that of Julien<sup>[9]</sup>, in which the main parameters considers are material density and grain size ( $D_{50}$ ). Their relation with the measured fall velocity is presented in Fig. 9. In this figure, it is very clear that for the range of settling velocities found in the littoral cell in study, the influence of density is not relevant. Furthermore grains with almost the same density have very different fall velocities and no tendency can be seen in the values. However, this does not mean that if a wider range of relative densities were compared some influence could not be detected. In this particular case, all the material has the same geological origin, thus the densities are very similar, but not so the fall velocities.

It is important to note that the material being studied is fine to medium sand with almost no cohesive material. The very small grain sizes found in these samples led to an analysis of the shape factor and the fall velocity of the grains and are shown in Fig. 10.

Since the formula of Julien<sup>[9]</sup> gives the best fit and density and shape are not relevant, only the  $D_{50}$  would seem to govern the settling velocity for the samples taken. The relation between  $D_{50}$  and fall velocity is shown in Fig. 11.

As expected, the  $D_{50}$  shows a clear tendency. Now it is easy to get a relation of the settling velocity as a function of the grain size considering that other parameters showed little relevance in the values of the phenomenon. Equation (3) is the best fit curve for the values measured at the laboratory.

$$Wf=a(b-\exp(-cD_{50})) \quad (3)$$

Table 4: Measured Wf of selected sand samples

ID	Wf m/s	
PL PM1		0.06010
PS PM2		0.06560
PB 1	0.05308	
PB 4A		0.04226
PL 5	0.05851	
PS 7	0.10351	
PL 9	0.07575	
PB 10		0.05175
PS C	0.06487	
PB F	0.02767	
PR F	0.14815	
PL F	0.01872	
PS F	0.00940	

Table 5: Average relative error of each fall velocity formula

Formula	%	Err
Ahrens <sup>[1]</sup>		12.35
Chang & Liou <sup>[2]</sup>		10.07
Cheng <sup>[3]</sup>		9.73
Hallermeier <sup>[7]</sup>		14.52
Jimenez & Madsen <sup>[8]</sup>		10.53
Julien <sup>[9]</sup>		8.77
Rubey <sup>[10]</sup>		13.79
Soulsby <sup>[11]</sup>		10.30
Van Rijn <sup>[12]</sup>		9.08

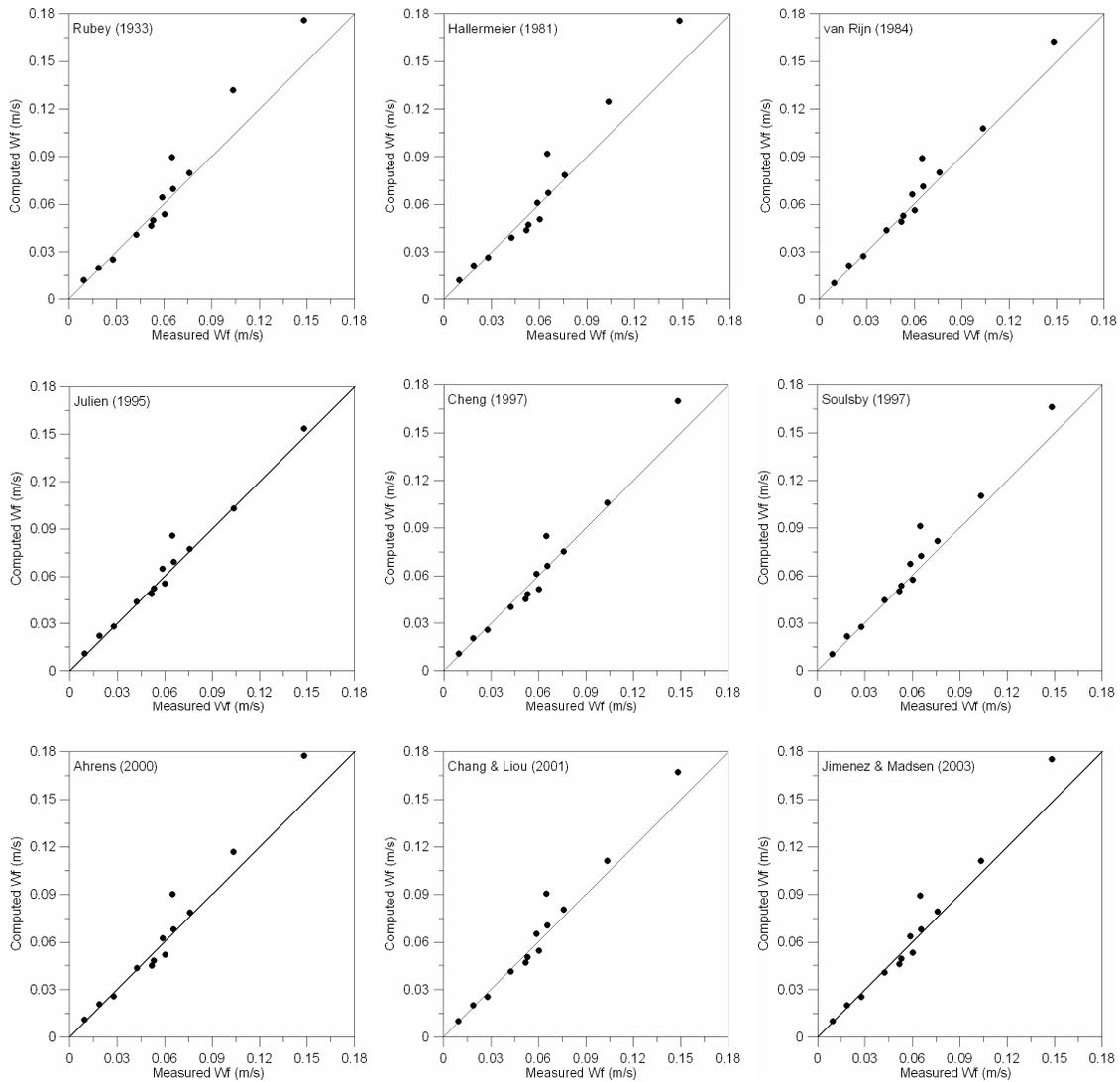


Fig. 8: Comparison of measured and theoretical (computed) settling velocities

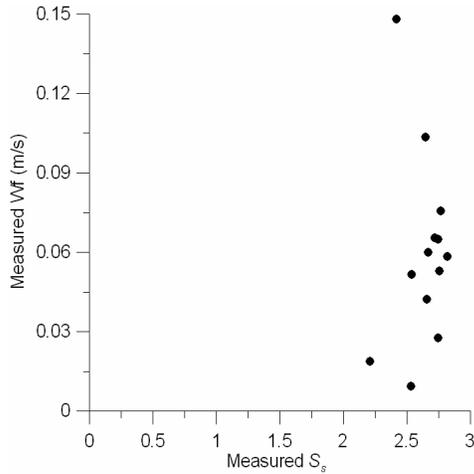


Fig. 9: Measured relative density versus fall velocity

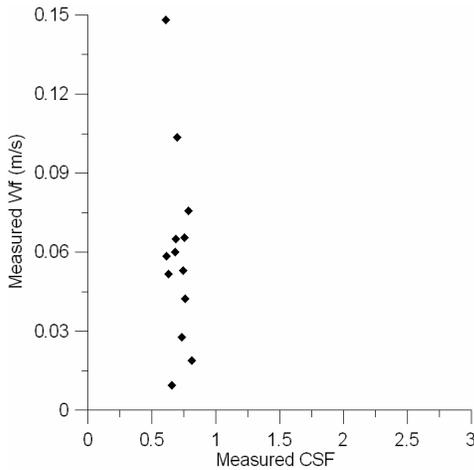


Fig.10: Measured Corey Shape Factor versus fall velocity

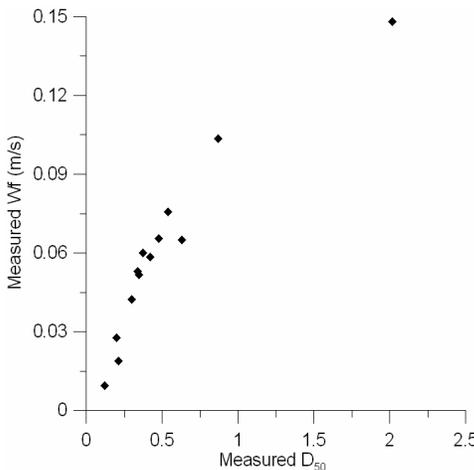


Fig.11: Measured  $D_{50}$  in mm versus fall velocity

Where  $a=0.17\text{m/s}$ ,  $b=0.93$  and  $c=1.23\text{mm}^{-1}$  are empirical parameters applicable to the area of interest shown in Fig. 1. As a very simple validation exercise, the settling velocity of the 60 samples is calculated with equation (3) and compared to that computed with Julien<sup>[9]</sup> and shown in Fig. 12.

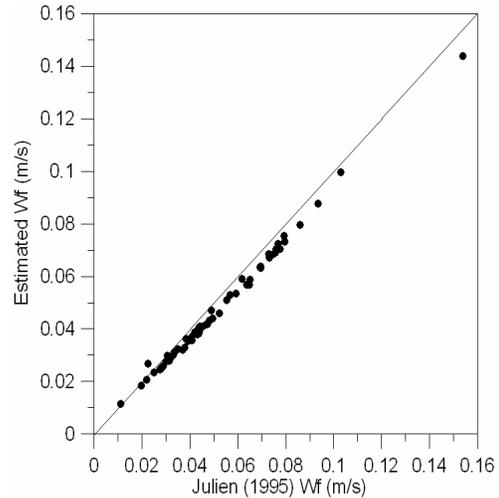


Fig.12:  $W_f$  estimated with ec (3) versus Julien (1995)

It can be seen that equation (3) slightly underestimates the velocities but the average relative error is only 8.69%. The approximation given by equation (3) is a very simple method to get the settling velocity for the material within the littoral cell studied and useful for engineering applications.

### CONCLUSIONS

The measurement of certain physical parameters of 60 sand samples, taken around Cancun, Quintana Roo, Mexico were analysed, measuring grain size distribution, relative density and Corey shape factor.

The relation between the properties measured and the settling velocity was investigated. For the material in the littoral cell studied, no relevant influence in velocity was found with density and shape but a very clear dependence on grain size was detected.

The fall velocity obtained from laboratory procedures was compared with a group of nine empirical formulations with that of Julien<sup>[9]</sup> agreeing best.

A simple relation to obtain settling velocity as a function of  $D_{50}$  was presented. This equation was

validated with the theoretical data calculated with Julien [9] giving acceptable results.

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