Kinematics of the Basic Mechatronic Module 3R of an Anthropomorphic Robot

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Abstract: The kinematics of the basic mechatronic module 3R of an anthropomorphic robot will be presented in this study, through an original geometric-analytical method, developed by the author. The advantages of the proposed new method are a great simplicity of calculations and calculation relations, intertwined with very high precision. The method is a strong one from a physical-mathematical point of view. There is a counter that must be set correctly to plus or minus 1, otherwise, all values and calculations are fast, accurate, direct, intuitive. The method has already been verified by the author with another original, older, trigonometric one and the results obtained by both methods are identical. If the trigonometric method already presented previously was a simple one, the geometric-analytical one proposed this time is even simpler in calculations and more precise, the effective work being to present the method and the calculation relations as well as the way they were deducted, but their use is very simple and fast.

Keywords: Kinematics, Mechatronic Module 3R, Anthropomorphic Robot, A Geometric-Analytical Method

Introduction

Robots have always fascinated us, but today we use them massively, in almost all industrial areas, especially where they work hard, repetitive and tiring, in toxic, chemical, radioactive environments, underwater, in the cosmos, in dangerous environments, on mined lands, in hard to reach areas, etc. It can be said once again that just as software and microchips have helped us to write various useful programs quickly and to implement them directly, so robotics has made our daily work much easier. Thanks to robots, automation is almost perfect today, the quality of the products is very high, the manufacturing price has dropped a lot, you can work in continuous fire, people have escaped hard work, tiring, repetitive, in toxic environments and can now deal other more important issues, such as design, scientific research, working only 5 days a week with high incomes and in the future also due to the massive implementation of increasingly modern robots with increased capabilities, man will reach the week of work only 4 days.

An even greater increase is expected in the number of specialized robots implemented in large factories and factories around the world (Fig. 1).

Fig. 1: Robotic line in the Mercedes factory

The initial problem was greatly diminished when the unions demanded the elimination of robots as enemies who kidnap people's jobs, but even today
unions sometimes negotiate the introduction of workers with robots in fully automated production lines and it is very dangerous to work side by side elbow people with robots, it was established at Mercedes for example that the workers are the ones who will practically assemble the Mercedes logo in the end, obviously after the whole car was created and assembled automatically only with robots.

Whether processing, translating, rotating, processing, painting, cutting, welding, assembling robots, the vast majority of robotic models used today in the factory even in complex robotic cells, use anthropomorphic robots with several degrees of freedom, able to develop high powers and torques, fast and dynamic, simple and cheap, economical, with sufficient stability and a suitable workspace for the necessary operations (Fig. 2-5).

![Total industrial automation with simple anthropomorphic robots](image1)

**Fig. 2:** Total industrial automation with simple anthropomorphic robots

![Automation with Tesla anthropomorphic robots](image2)

**Fig. 3:** Automation with Tesla anthropomorphic robots
Fig. 4: An assembly line production of some new cars. Automated welding of the car body on the production line; the robotic arm on the car production line is working.

Fig. 5: BMW 5 Series CAR FACTORY

As we have shown in other previous works, anthropomorphic robots all have a basic spatial structure (Fig. 6), which can be studied more simply in plan (Fig. 7) if we eliminate the rotational module that rotates the flat platform in various directions (thus transforming the flat base motion into a spatial motion).

About 90% of anthropomorphic robots are used worldwide today because they can be designed, built and implemented easier, cheaper and are very reliable (Anderson, 1997; CEUP, 2018; García, 2020; Rana, 2020; Garfo et al., 2020; Kumar and Sreenivasulu, 2019; Mishra and Sarawagi, 2020; Welabo and Tesfamariamr, 2020; Antonescu and Petrescu, 1985; 1989; Antonescu et al., 1985a; 1985b; 1986; 1987; 1988; 1994; 1997; 2000a; 2000b; 2001; Aversa et al., 2017a; 2017b; 2017c; 2017d; 2016a; 2016b; 2016c; 2016d; Ayieie, 2020; Brewer, 1991; Chilukuri et al., 2019; Cao et al., 2013; Dong et al., 2013; Saheed et al., 2019; Riman, 2019; Matthews and Yi, 2019; Dwivedi et al., 2019a; 2019b; Eremia, 2020; Franklin, 1930; Hanrahan, 2014; He et al., 2013; Hertel, 2017; Komakula, 2019; Langston, 2015; 2016; Lee, 2013; Lin et al., 2013; Liu et al., 2013; Padula and Perdereau, 2013; Perumaal and Jawahar, 2013; Petrescu, 2011; 2012; 2019a-v; 2020a-g; Petrescu and Petrescu, 2019a-f; 1995a; 1995b; 1997a-c; 2000a-b; 2002a-b; 2003; 2005a-e; 2011a-c; 2012a-b; 2013a-e; 2014a-h; 2016a-c; 2020; Petrescu et al., 2007; 2009; 2016; 2017a-ak;...
2018a-w; 2020; Petrescu and Calautit, 2016a-b; Dekkata and Yi, 2019; Fahim et al., 2019; El Hassouni et al., 2019; Rimam, 2018; Nacy and Nayif, 2018; Kortam et al., 2018; Welch and Mondal, 2019; Eissa et al., 2019; Younes et al., 2019; Svensson et al., 2004; Rahman, 2018; Richmond, 2013; Kisabo et al., 2019a; 2019b; Kisabo and Adeimpe, 2019; Kosambe, 2019a; 2019b; 2019c; 2019d; Sharma and Kosambe, 2020; Oni and Jha, 2019; Chaudhary and Kumar, 2019; de Lima et al., 2019; Babu et al., 2019; 2020; de Mota Siqueira et al., 2020; Tumino, 2020; Mishra, 2020a; 2020b; Brischetto and Torre, 2020; Vladescu, 2020).

**Fig. 6:** The basic spatial structure

**Fig. 7:** The basic planar structure driven in reverse kinematics by a crank so that the exit point performs a complete rotation
Materials and Methods

The structure of Fig. 8 consists of two elements connected to each other by a fifth-class flat rotational coupling at point C and having at the ends one or even two other fifth-class rotational couplings.

Usually the outer coupling, from point B, of entry, is also a flat rotation torque as the inner one from C and in D it can only be a working point of the respective manipulator or robot, or another external coupling can be caught, to which the defector is connected, i.e., the final device of the robot: It can be a gripper, i.e., a gripping, gripping and handling device, it can be a welding electrode, it can be a paint gun, a soldering iron, a any working device, or an arm may be placed to extend the working capabilities of the robot; In point D, therefore, there may be no more couplings, there may be a fifth-class plane of rotation as well as those in points B and C, or there may be another coupling, for example spatial; if the module consisting of the two arms 2 and 3 is used and/or only at, any mechanism, the simplest being the planar quadrilateral mechanism, or the articulated quadrilateral mechanism, which has the kinematic scheme shown in the figure at the top left, then the module is mandatory from the studied right will have a kinematic torque of rotation, plane, of the fifth class and in point D, the module 2-3 having in this case the name of structural group, of type: Dyad 3R. This case being the most general (complete) we will start in this study with it and we will study the direct and indirect kinematics of this module (dyad 3R) that we saw that it can be used, generalize to the vast majority of anthropomorphic robots. The classic robots are of the anthropomorphic type, i.e., serial robots and most of them have rotational movements, the drives being made with stepping actuators (motors). All anthropomorphic structures are based on a 3R robot as can be seen in Fig. 6.

The idea is to greatly simplify the calculations and relationships (including the classical methods used), moving from spatial study to different planes. It can be seen that if we separate the rotational movement $\phi_{10}$ from the basic plane $x_0O_0y_0$, decoupling it from the other rotational movements $\phi_{20}$ and $\phi_{30}$ we arrive at our module, where the torque in B is denoted here by $A$ ($O_2$, being a point constructive), the coupling in C is denoted here with $B$ ($O_3$ being a constructive point) and the coupling or working point of the end-effector in D is denoted here with $M$. This idea greatly simplifies the classical spatial calculations, especially those for reverse kinematics, because this is the most difficult, presented in the course of SMMSP) transforming them into plane calculations (follow the complete method in the course "Mechatronics-SSP").

The proposed study module will be in this topic (the complete, general plan of a structural group, type: Dyad 3R, or dyad RRR; see the kinematic diagram of the module in the figures). We always know (give) the constant lengths of the two elements of the module: $l_2$ and $l_3$, the positions of the outer coupling, input B ($x_B$, $y_B$, $z_B$), in our case, with flat treatment B ($x_B$, $y_B$). In direct kinematics, the simpler position angles $\phi_2$ and $\phi_3$ are also known and the positions of the point (outer coupling) D, i.e., ($x_D$, $y_D$), are required. In inverse kinematics (our topic), the positions of the defector D, i.e., $x_D$, $y_D$, are also known, imposed (imposed) and the position angles $\phi_2$ and $\phi_3$ are required (to be determined) Fig. 9.

![Fig. 8: The basic planar anthropomorphic structure](image-url)
If at the old presented method, the trigonometric one, the angles $FI_2$ and $FI_3$ were determined first of all and then with their help the scalar parameters of point $C$ can be calculated (Fig. 9, Eq. 1), by the newly proposed method, geometric-analytical is determined directly the scalar coordinates of the point (couples) $C$ (Eqs. 2 and 3) and then to calculate the two angles $FI_2$ and $FI_3$ easily now that all the scalar coordinates of all the couples of the mechatronic module ($B$, $C$ and $D$) are known:

\[
\begin{align*}
  x_c &= x_a + l_1 \cdot \cos \varphi_2 \\
  y_c &= y_a + l_2 \cdot \sin \varphi_2 \\
  \text{or} \\
  x_c &= x_d - l_3 \cdot \cos \varphi_3 \\
  y_c &= y_d - l_3 \cdot \sin \varphi_3
\end{align*}
\]

(1)

\[
\begin{align*}
  x_c &= x_a + l_1 \cdot \cos \varphi_2 \Rightarrow \cos \varphi_2 = \frac{x_c - x_a}{l_2} \\
  y_c &= y_a + l_2 \cdot \sin \varphi_2 \Rightarrow \sin \varphi_2 = \frac{y_c - y_a}{l_2} \\
  \Rightarrow \varphi_2 &= \arccos \left( \cos \varphi_2 \right) \cdot \text{sign} \left( \sin \varphi_2 \right) \\
  \Rightarrow \varphi_2 &= \arccos \left( \frac{x_c - x_a}{l_2} \right) \cdot \text{sign} \left( \frac{y_c - y_a}{l_2} \right) \\
  x_d &= x_c + l_3 \cdot \cos \varphi_3 \Rightarrow \cos \varphi_3 = \frac{x_d - x_c}{l_3} \\
  y_d &= y_c + l_3 \cdot \sin \varphi_3 \Rightarrow \sin \varphi_3 = \frac{y_d - y_c}{l_3} \\
  \Rightarrow \varphi_3 &= \arccos \left( \cos \varphi_3 \right) \cdot \text{sign} \left( \sin \varphi_3 \right) \\
  \Rightarrow \varphi_3 &= \arccos \left( \frac{x_d - x_c}{l_3} \right) \cdot \text{sign} \left( \frac{y_d - y_c}{l_3} \right)
\end{align*}
\]

(2)

\[
\begin{align*}
  \varphi_2 &= \arccos \left( \frac{x_c - x_a}{l_2} \right) \cdot \text{sign} \left( \frac{y_c - y_a}{l_2} \right) \\
  \varphi_3 &= \arccos \left( \frac{x_d - x_c}{l_3} \right) \cdot \text{sign} \left( \frac{y_d - y_c}{l_3} \right)
\end{align*}
\]

(3)

In other words, it is much simpler and more precise to first determine the scalar coordinates of the coupling $C$ ($x_C$ and $y_C$) and then the angles $FI_2$ and $FI_3$ (with Eq. 3), than to calculate first the angles and then the coordinates of point $C$ (with Eq. 1).

### Results and Discussion

In other words, for the inverse kinematics of the module (at which the constant lengths of the two elements, 2 and 3, i.e., $l_2$ and $l_3$, but also the positions, speeds and accelerations of the inputs, torques and/or external points, $B$ and $D$ are known $x_B, y_B, x_D, y_D, x_F, y_F, x_{F1}, y_{F1}, x_{F2}, y_{F2}, x_{F3}, y_{F3}$): Are required to determine the position angles of the two elements, 2 and 3 and their derivatives: $\varphi_2, \varphi_3, \omega_2, \omega_3, \epsilon_2, \epsilon_3$. The values of the angles $FI_2$ and $FI_3'$ depending on the input value of the angle of the crank 1, $FI_1$, all given in degrees [deg], can be traced in the diagram in Fig. 10.

The angular velocity hodograph $w_3$ as a function of $w_2$ can be traced in Fig. 11.

The angular velocities $w_2$ and $w_3$ vary depending on the entry angle of the crank, $FI_1$, according to the graphs in Fig. 12.

Similarly, the graph of the variation of the angular accelerations of elements 2 and 3 is obtained depending on the position of the angle $FI_1$ (Fig. 13).

In the points of intersection of the two graphs, practically there are equal the angular velocities in Fig. 12 and the angular accelerations for the situation in Fig. 13. If
at speeds there are two points where the angular velocities of the two different elements are equal, at accelerations, there are three such situations in which the angular accelerations of the two different elements become equal.

**Fig. 10:** The values of the angles $\theta_2$ and $\theta_3'$ depending on the input value of the angle of the crank $\theta_1$.

**Fig. 11:** The angular velocity hodograph $w_3$ as a function of $w_2$. 
The new method for determining positions in inverse kinematics, based on analytical geometry, will be briefly set out in the Annex.

Conclusion

The paper briefly presents the results obtained in the inverse kinematics of an anthropomorphic mechatronic basic plane module, when its exit point, end-effector describes a complete circle (whose given coordinates, known, were imposed in the work with a crank, which can be imaginary).

The graphs of the positions, velocities and angular accelerations of the elements noted with 2 and 3 of the composition of the basic plane mechatronic structure are presented.

The calculations were performed consecutively by two different methods, an older trigonometric one and a new geometro-analytical one, in both situations the results obtained being identical.

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Ethics

This article is original and contains unpublished material. Author declares that are not ethical issues and no conflict of interest that may arise after the publication of this manuscript.

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Source of Figures

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Fig. 2: http://www.automaticbagpackingmachine.com/sale-9566254-industrial-fully-automated-welding-production-line-plc-control-for-car-industry.html

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Fig. 4: https://www.welding-carbody-automotive-production-line-long-format-wide-frame-assembly-line-production-new-car-automated-welding-car-image133841638

Fig. 5: https://www.youtube.com/watch?v=9fjnMJauGwU

Fig. 6-13: Made by author

Appendix

The new, geometric-analytical equations used are the following (system 4):
where \( a \) is the length of element 2, \( b \) represents the length of element 3, \( c \) is the variable length of the segment joining the two input couplings in the plane mechatronic module, the input torques of the mechatronic module being denoted by \( B \) and \( D \) and its inner coordinates being denoted with \( C \); the Cartesian coordinates of the input torques \( B \) and \( D \) are known, while the coordinates of the inner point \( C \) must be determined. With \( h \) the difference in height between the entry points in mode \( B \) and \( D \) was noted, while with \( d \) the horizontal difference between the same input couplings \( B \) and \( D \) was noted, according to the last two relations in the system below. The \( p \) parameter is a negative or positive sign, a counter that can take the value \(-1 \) or \( 1 \).

The equations of the older trigonometric method are mentioned in the system (5):

\[
\begin{align*}
\cos \varphi &= \frac{y_{b} - y_{a}}{l} \quad \text{cos} \varphi &= \frac{x_{b} - x_{a}}{l} \quad \varphi = \text{semn} (\sin \varphi) \cdot \arccos (\cos \varphi) \Rightarrow \\
\Rightarrow \varphi &= \text{semn} \left( \frac{y_{b} - y_{a}}{l} \right) \cdot \arccos \left( \frac{x_{b} - x_{a}}{l} \right) \\
\cos B &= \frac{l_{1}^{2} + l_{2}^{2} - l_{3}^{2}}{2 \cdot l_{1} \cdot l_{2}} \Rightarrow B = \arccos (\cos B) = \arccos \left( \frac{l_{1}^{2} + l_{2}^{2} - l_{3}^{2}}{2 \cdot l_{1} \cdot l_{2}} \right) \Rightarrow \\
\cos D &= \frac{l_{1}^{2} + l_{3}^{2} - l_{2}^{2}}{2 \cdot l_{1} \cdot l_{3}} \Rightarrow D = \arccos (\cos D) = \arccos \left( \frac{l_{1}^{2} + l_{3}^{2} - l_{2}^{2}}{2 \cdot l_{1} \cdot l_{3}} \right) \Rightarrow \\
\alpha_{2} &= \frac{(x_{b} - x_{a}) \cdot \cos \varphi_{2} + (y_{b} - y_{a}) \cdot \sin \varphi_{2}}{l_{2} \cdot \sin (\varphi_{3} - \varphi_{2})} = \frac{l_{2}}{l_{1} \cdot \sin (\varphi_{3} - \varphi_{2})} \cdot \alpha_{1} \\
\alpha_{1} &= \frac{(x_{b} - x_{a}) \cdot \cos \varphi_{1} + (y_{b} - y_{a}) \cdot \sin \varphi_{1}}{l_{1} \cdot \sin (\varphi_{3} - \varphi_{1})} = \frac{l_{1}}{l_{1} \cdot \sin (\varphi_{3} - \varphi_{1})} \cdot \alpha_{1} \\
e_{2} &= \frac{(x_{b} - x_{a}) \cdot \cos \varphi_{2} + (y_{b} - y_{a}) \cdot \sin \varphi_{2} + l_{1} \cdot \omega_{2} \cdot \cos (\varphi_{2} - \varphi_{1})}{l_{1} \cdot \sin (\varphi_{3} - \varphi_{2})} \\
e_{1} &= \frac{(x_{b} - x_{a}) \cdot \cos \varphi_{1} + (y_{b} - y_{a}) \cdot \sin \varphi_{1} + l_{1} \cdot \omega_{1} \cdot \cos (\varphi_{2} - \varphi_{1})}{l_{1} \cdot \sin (\varphi_{3} - \varphi_{1})}
\end{align*}
\]