

Analysis of the Tunnel-Support Interaction Through a Probabilistic Approach

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Abstract: The analysis of the behavior of a tunnel requires the evaluation of numerous parameters, relative to the ground and the support structure and which are generally only known with a certain degree of approximation. The uncertainty of these parameters is reflected on the results of the calculation, that is, on the state of stress in the support structure. In order to be able to manage this uncertainty on the representative ground and support structure parameters in a rational way, it is useful to resort to a probabilistic type analysis, making use of the Monte-Carlo method and to simple analytical methods for an analysis of the tunnel-support structure interaction. A calculation procedure of probabilistic type that is able to establish representative values of the maximum moment and of the associated normal force in the support structure is presented in this study. This procedure can be used to verify the suitability of the support structure to sustain the induced loads.

Keywords: Tunnel Support, Convergence-Confinement Method, Deep Circular Tunnel, Loads on the Support, Uncertainty of the Parameters, Probability Distribution of the Parameters, Cumulative Frequency, Monte-Carlo Method

Introduction

The analysis of the behavior of a tunnel and the dimensioning of the support structures necessary to guarantee its stability require the knowledge of numerous parameters that are essential for the calculation. These parameters make it possible to describe the ground in which the tunnel is excavated, the entity of the pre-existing stress state at the depth of the tunnel and the geometrical and mechanical characteristics of the support structure that is to be installed. Unfortunately, many of these parameters are only known with a certain approximation, also because they can vary along the tunnel stretch (Karakostas and Manolis, 2000; Oreste, 2005a; Špacková *et al.*, 2013).

For this reason, reference to calculation parameters established according to a deterministic approach could lead to serious problems. Instead, a probabilistic approach allows one to have precise indications on the uncertainty of the calculation parameters and to obtain results associated to the requested level of reliability (Lü and Low, 2011; Fellin *et al.*, 2010; Oreste, 2005b; Kalamaras, 1997; Guarascio *et al.*, 2007a; 2007b; 2013; Lombardi *et al.*, 2013). In other words, the probabilistic

approach is able to deal with the in situ uncertainty of the calculation parameters and of the obtained results in a rational way.

In order to be able to conduct a probabilistic type analysis, it is generally necessary to refer to the Monte-Carlo method, which allows the single uncertain calculation parameters to be extracted randomly, once their probabilistic distribution has been defined (type of distribution, mean value and standard deviation for each parameter considered uncertain) (Oreste, 2006; Tonon *et al.*, 2000).

Moreover, as numerous calculations are necessary for each series of extracted parameters (random vector), it is necessary to adopt simple calculation methods, of an analytical type, that lead to a closed solution which can be reached in a reasonable time (Osgoui and Oreste, 2007; Oreste, 2009a; 2013). Numerical type calculation methods (Do *et al.*, 2013; 2014a; 2014b), which are usually slower and more complex, do not in fact lend themselves to probabilistic type analyses in the tunneling sector.

The convergence-confinement method (Rechsteiner and Lombardi, 1974; Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; AFTES, 1993; Panet,

1995; Panet *et al.*, 2001) and the Einstein and Schwartz (1979) method are two of the most commonly used analytical methods in the field of tunneling. The former allows the stresses and strains that develop around a deep circular tunnel to be analyzed as well as the interaction between the tunnel and the support structure (Oreste, 2003a; 2003b; 2009b): The value of the radial stress acting at the extrados of the support structure is an important result that can be obtained. The latter method allows the maximum moment acting within the support and the normal force associated to it to be simply obtained through the application of a certain load to the support structure.

Once the probabilistic distribution of the maximum moment values and the associated normal force have been obtained, it is possible to proceed with a verification of the capacity of the hypothesized support to absorb its internal induced stresses.

In this study, the convergence-confinement method and the techniques necessary to proceed with a study of the interaction between a tunnel and a support structure are presented first, in order to evaluate the load acting on the support structure. Then, a technique is indicated that can be used to be able to determine the maximum moment and the associated normal force, once the load acting on the support structure is known, through the Einstein and Schwartz method.

Finally, the modalities that can be used to treat the analyses of the behavior of a support structure in probabilistic terms, through the use of the Monte-Carlo method, are given. In particular, the modalities that can be used, at the end of the probabilistic analyses, to obtain the values of the maximum moment and the normal force, from which the maximum stresses in the support structure are obtained, are indicated. These values could then be compared with the strength of the material that makes up the support in order to have indications on whether the hypothesized support structure is adequate, lacking or excessive.

Materials and Methods

The convergence-confinement method allows an analysis to be made of the interaction between a tunnel and a support, on the basis of the following hypotheses: Circular and deep tunnel (depth of the tunnel axis from the surface greater than 10-12 times the tunnel radius R); homogeneous and isotropic ground; constant and isotropic lithostatic stress p_0 ($K_0 = 1$) around the tunnel (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; Panet, 1995; Oreste, 2009b).

The convergence-confinement curve is the relation that connects the radial displacements of the tunnel wall u_R to the pressure applied inside the tunnel σ_R . For elastic behavior of the ground around the tunnel, the

following is obtained (Rechsteiner and Lombardi, 1974; Ribacchi and Riccioni, 1977; Panet, 1995):

$$u_R = \frac{1+v}{E} \cdot (p_0 - \sigma_R) \cdot R \quad (1)$$

Where:

E = The elastic modulus of the ground

v = The Poisson ratio of the ground

In the case of elastic-plastic behavior, a plastic zone develops around the tunnel when the internal applied pressure σ_R is below σ_{Rpl} (radial stress at the border between the plastic zone and the zone with elastic behavior) and when the latter is greater than 0.

Therefore, for $\sigma_R \leq \sigma_{Rpl}$ and $\sigma_{Rpl} \geq 0$, the relation between u_R and σ_R is no longer obtained using Equation 1, but from the following Equation 2, considering the Mohr-Coulomb strength criterion as being valid (Ribacchi and Riccioni, 1977):

$$u_R = \frac{D}{N_{\phi,r} + N_\psi} \cdot R^{N_{\phi,r}} + \frac{F}{N_\psi + 1} \cdot R + \left(u_{Rpl} \cdot R_{pl}^{N_\psi} - \frac{D}{N_{\phi,r} + N_\psi} \cdot R_{pl}^{N_{\phi,r} + N_\psi} - \frac{F}{N_\psi + 1} \cdot R_{pl}^{N_\psi + 1} \right) \cdot R^{-N_\psi} \quad (2)$$

Where:

$$N_{\phi,p} = \frac{1 + \sin \phi_p}{1 - \sin \phi_p} \quad \sigma_{c,p} = \frac{2 \cdot c_p \cdot \cos \phi_p}{1 - \sin \phi_p}$$

$$N_{\phi,r} = \frac{1 + \sin \phi_r}{1 - \sin \phi_r} \quad \sigma_{c,r} = \frac{2 \cdot c_r \cdot \cos \phi_r}{1 - \sin \phi_r} \quad N_\psi = \frac{1 + \sin \psi}{1 - \sin \psi}$$

c_p and c_r = The ground peak and residual cohesion

ϕ_p and ϕ_r = The ground peak and residual friction angle

Ψ = The dilatancy (dilatancy is an angle that can vary between 0 and the residual friction angle of the ground)

$$D = \frac{(1-v^2)}{E} \cdot$$

$$\left\{ \left(\sigma_R + \frac{\sigma_{c,r}}{(N_{\phi,r} - 1)} \right) \cdot \frac{1}{R^{(N_{\phi,r} - 1)}} \cdot \left(1 - N_\psi \cdot \frac{v}{1-v} \right) + \left(\sigma_R + \frac{\sigma_{c,r}}{(N_{\phi,r} - 1)} \right) \cdot \frac{1}{R^{(N_{\phi,r} - 1)}} \cdot N_{\phi,r} \cdot \left(N_\psi - \frac{v}{1-v} \right) \right\}$$

$$F = \frac{(1-v^2)}{E} \cdot$$

$$\left\{ \left(-\frac{\sigma_{c,r}}{(N_{\phi,r} - 1)} - p_0 \right) \cdot \left(1 - N_\psi \cdot \frac{v}{1-v} \right) + \left[\left(-\frac{\sigma_{c,r}}{(N_{\phi,r} - 1)} \right) \cdot N_{\phi,r} + \sigma_{c,r} - p_0 \right] \cdot \left(N_\psi - \frac{v}{1-v} \right) \right\}$$

$$u_{Rpl} = \frac{1+\nu}{E} \cdot (p_0 - \sigma_{Rpl}) \cdot R_{pl}$$

$$\sigma_{Rpl} = \frac{2 \cdot p_0 - \sigma_{c,p}}{(N_{\phi,p} + 1)}$$

$$R_{pl} = R \cdot \left[\frac{(N_{\phi,r} - 1) \cdot \sigma_{Rpl} + \sigma_{c,r}}{(N_{\phi,r} - 1) \cdot \sigma_R + \sigma_{c,r}} \right]^{\frac{1}{(N_{\phi,r} - 1)}}$$

Therefore, if $\sigma_{Rpl} < 0$, the convergence-confinement curve can be obtained from Equation 1, which expresses a linear trend of u_R as σ_R varies from p_0 to 0. Instead, if $\sigma_{Rpl} \geq 0$, the convergence-confinement curve is expressed by Equation 1 for σ_R varying from p_0 to σ_{Rpl} and from Equation 2, for σ_R varying from σ_{Rpl} to 0.

The interaction between the tunnel and the support structure can be studied by overlapping the support reaction line onto the convergence-confinement curve of the tunnel. This reaction line is expressed by the following σ_R - u_R relation (Panet, 1995; Oreste, 2003a):

$$\sigma_R = k_{sup} \cdot (u_R - u_{Ro}) \quad (3)$$

where, k_{sup} is the stiffness of the support; in the case of a continuous shotcrete lining, the stiffness of the support is given, for example, by the following relation (Hoek and Brown, 1980):

$$k_{sup} = \frac{E_{sup}}{(1 + \nu_{sup})} \cdot \frac{R^2 - (R - t_{sup})^2}{(1 - 2 \cdot \nu_{sup}) \cdot R^2 + (R - t_{sup})^2} \cdot \frac{1}{R} \quad (4)$$

E_{sup} = The elastic modulus of the lining material

ν_{sup} = The Poisson ratio of the lining material

t_{sup} = The thickness of the lining

u_{Ro} = The radial displacement of the tunnel that has already developed at the point in which the support has been installed

According to Vlachopoulos and Diederichs (2009), this value can be estimated in function of the maximum displacement u_{Rmax} that occurs at a long distance from the excavation face, in function of the distance d from the excavation face at which the support is installed and in function of the value of the plastic radius $R_{pl}(\sigma_{Req})$ for a pressure σ_R equal to σ_{Req} (the final pressure acting on the support structure):

$$u_{Ro} = u_{Rmax} \cdot \left[1 - \left(1 - \frac{1}{3} \cdot e^{-0.15 \cdot \frac{R_{pl}(\sigma_{Req})}{R}} \right) \cdot e^{\left[\frac{-3 \cdot d}{2 \cdot R_{pl}(\sigma_{Req})} \right]} \right] \quad (5)$$

As both u_{Rmax} and σ_{Req} can be obtained from the intersection of the convergence-confinement curve with the reaction line of the support and they therefore depend

on the u_{Ro} , it is necessary to have an iterative procedure that quickly converges to a final value, starting from the initial condition $u_{Ro} = 0$ (Fig. 1). At the end of the iterative procedure, a value of the maximum displacement of the tunnel wall u_{Rmax} (at a long distance from the excavation face) and a value of the final load acting on the support structure σ_{Req} are obtained. This load is very useful for verifying whether the initially hypothesized support structure can be considered adequate.

In order to proceed with this verification, it is necessary to refer to the maximum bending moment M_{max} and to the associated normal force N induced inside the support. It is possible to estimate the maximum compression stress and the maximum tensile stress (if it exists) inside the support structure using this previous couple of values (M_{max} and N).

M_{max} and the associated N that act inside the tunnel supports, can be obtained adopting the Einstein and Schwartz (1979). This method belongs to the analytical method category that does not require a numerical solution and it is relatively simple to carry out the calculations. It considers the medium around the lining as being elastic, homogeneous and isotropic and of being of infinite extension, with an initial stress (a different value of the vertical stress and of the horizontal stress) equal to the lithostatic stress. The tunnel lining is treated as an elastic ring, with a normal stiffness and a bending stiffness, which are described by the following dimensionless parameters C^* and F^* , respectively (Einstein and Schwartz, 1979):

$$C^* = \frac{E \cdot R \cdot (1 - \nu_{sup}^2)}{E_{sup} \cdot A_{sup} \cdot (1 - \nu^2)} \quad F^* = \frac{E \cdot R^3 \cdot (1 - \nu_{sup}^2)}{E_{sup} \cdot I_{sup} \cdot (1 - \nu^2)} \quad (6)$$

Where:

A_{sup} = The area of the lining section: $A_{sup} = t_{sup} \cdot l$

I_{sup} = The inertia moment of the lining section: $I_{sup} = \frac{t_{sup}^3 \cdot l}{12}$

Let us consider the support-tunnel wall interface case, which supposes relative displacement without any limitation. It is possible to obtain M_{max} and the associated N through the following simple relation (Einstein and Schwartz, 1979), under the conservative hypothesis that the load σ_{Req} obtained from the convergence-confinement method represents the radial stress acting at the extrados of the support in correspondence to the crown zone:

$$M_{max} = \alpha \cdot \sigma_{Req} \cdot R^2 \cdot \left[\frac{1}{2} \cdot (1 - K_0) \cdot (1 - 2 \cdot a_2^*) \right] \quad (7)$$

$$N = \alpha \cdot \sigma_{Req} \cdot R \cdot \left[\frac{1}{2} \cdot (1 + K_0) \cdot (1 - a_0^*) \right] \quad (8)$$

Where:

K_0 = The lateral thrust coefficient in lithostatic conditions (ratio between the horizontal stresses and the vertical ones)

a_0^* , a_2^* and α = Dimensionless coefficients:

$$a_0^* = \frac{C^* \cdot F^* \cdot (1-\nu)}{C^* + F^* + C^* \cdot F^* \cdot (1-\nu)} \quad a_2^* = \frac{(F^* + 6) \cdot (1-\nu)}{2 \cdot F^* \cdot (1-\nu) + 6 \cdot (5-6 \cdot \nu)}$$

$$\alpha = \frac{1}{2 - K_0 - \frac{1}{2} \cdot a_0^* \cdot (K_0 + 1) + 3 \cdot a_2^* \cdot (K_0 - 1)}$$

In order to be able to obtain the two fundamental values of M_{max} and the associated N , it is necessary to refer to a series of parameters, which are usually known with a certain approximation. In particular, the following characteristic parameters of the ground (c_p , c_r , ϕ_p , ϕ_r , E), the lithostatic vertical stress p_0 , the coefficient K_0 and the characteristic parameters of the support structure (E_{sup} , t_{sup}) can present a certain variability and it is often difficult to attribute a representative parameter for all of the conditions that can be encountered during the excavation of a tunnel.

It would be more appropriate to proceed with a probabilistic study, in which the parameters of the problem are represented by their intrinsic uncertain value; a normal distribution is generally adopted. This distribution is generally interrupted at $\pm 3 \cdot \sigma$ of the mean value μ of the distribution (where μ is the mean value and σ the standard deviation of the population). In this way, 0.27% of the values outside the $\mu \pm 3 \cdot \sigma$ interval are excluded. Therefore, if a variability interval of an uncertain parameter ($\beta \pm \beta \cdot \mu$, with β expressed in percentage terms) is identified, it is possible to evaluate the standard deviation σ of the probabilistic distribution of the population in the following way: $\sigma = (\beta \cdot \mu) / 3$. Once the probabilistic distributions of each parameter of the problem have been determined (through μ and σ), it is possible to proceed with a random extraction of these parameters using the Monte-Carlo method (Karakostas and Manolis, 2000; Oreste, 2005a; Fellin *et al.*, 2010; Oreste, 2006; Tonon *et al.*, 2000).

The Monte-Carlo procedure, applied to the convergence-confinement method, allows a value of the σ_{Req} load to be obtained for each extracted vector; each vector is represented by the set of the single values extracted for each probabilistic parameter that characterizes the problem. The procedure therefore leads to the establishment of a sample of σ_{Req} values; the number of values of the sample is considered sufficient when the stabilization of the mean value and/or standard deviation of the sample is verified through an evaluation of the variability of the mean value and/or standard deviation of the σ_{Req} population. For this purpose, the

Student probabilistic distribution should be considered to evaluate the variability interval of the mean value and the distribution of X^2 for an evaluation of the variability interval of the standard deviation:

$$\bar{x} - t_{1-\frac{\alpha}{2}, n-1} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{1-\frac{\alpha}{2}, n-1} \cdot \frac{s}{\sqrt{n}} \quad (9)$$

Where:

$t_{1-\frac{\alpha}{2}, n-1}$ = The value of the Student distribution, for n-1 degree of freedom, for which the integral between $-\infty$ and $t_{1-\frac{\alpha}{2}, n-1}$ is equal to $1-\alpha/2$

\bar{x} = The mean value of the sample

s = The standard deviation of the sample

α = The reliability of the estimation of the variability interval

n = The samples size

μ = Is the mean value of the population (unknown):

$$\chi_{\frac{\alpha}{2}, n-1}^2 < \frac{(n-1) \cdot s^2}{\sigma^2} < \chi_{1-\frac{\alpha}{2}, n-1}^2 \quad (10)$$

Where:

$\chi_{\frac{\alpha}{2}, n-1}^2$ = The value of the X^2 distribution, for n-1 degree of freedom, for which the integral between $-\infty$ and $\chi_{\frac{\alpha}{2}, n-1}^2$ is equal to $\alpha/2$

$\chi_{1-\frac{\alpha}{2}, n-1}^2$ = The value of the X^2 distribution, for n-1 degree of freedom, for which the integral between $-\infty$ and $\chi_{1-\frac{\alpha}{2}, n-1}^2$ is equal to $1-\alpha/2$

σ = The standard deviation of the population (unknown)

From which the following is obtained:

$$\sqrt{\frac{(n-1) \cdot s^2}{\chi_{1-\frac{\alpha}{2}, n-1}^2}} < \sigma < \sqrt{\frac{(n-1) \cdot s^2}{\chi_{\frac{\alpha}{2}, n-1}^2}}$$

As the number of values of σ_{Req} in the sample grows, the estimation interval of the mean value and of the standard deviation of the population of σ_{Req} decreases. A sample that allows the semi-amplitude of the variability interval of the mean value below 1% of the mean of the sample to be obtained with a reliability of 99% and the semi-amplitude of the variability interval of the standard deviation of the population below 2% of the standard deviation of the sample to be obtained with a reliability of 95%, may be considered sufficient.

The thus obtained σ_{Req} sample can then be used to obtain a set of couples of M_{max} and N values through the Einstein and Schwartz (1979) method. Together with the σ_{Req} sample, other samples of uncertain parameters that intervene in the calculation with the Einstein and Schwartz method can be extracted with the Monte-Carlo method; for example, the elastic modulus of the ground E , the elastic modulus of the material that constitutes the support E_{sup} , the thickness of the support t_{sup} and the value of the coefficient K_0 . Again in this case, each single vector of the uncertain parameters produces a couple of values of M_{max} and N . Therefore, the final result is a sample of couples of values of M_{max} and N . This sample can be treated in probabilistic terms in order to identify the value of M_{max} that corresponds to a probability, for example, equal to 99%, that no values of the maximum moment will occur in the support structure above this value. Then, by analyzing the associated N values of all the values of M_{max} close to that probability, it is possible to obtain a probabilistic distribution of the values of N . In this case, the values of N at the ends of the extracted cumulative probabilistic distribution are of particular interest; for example, the values of N relative to a probability of 5 and of 95%, which can be used to identify the values of the maximum compression stress and of the maximum tensile stress (if it exists), in order to compare them with the relative compression and tensile strengths of the material that makes up the support structure.

Results

The probabilistic procedure explained in the previous section has been applied to the case of a circular tunnel, with a 4.5 m radius, excavated in a rock mass with a GSI = 35 (Marinos *et al.*, 2005; Hoek *et al.*, 2013; Marinos and Hoek, 2000; Cai *et al.*, 2004), an uniaxial compression strength of the intact rock σ_c equal to 80 MPa and a Hoek and Brown strength parameter of the intact rock m_i equal to 18 (Hoek *et al.*, 2002; Hoek and Brown, 1997; Hoek, 2007). On the basis of these parameters, it has been possible to make an estimation of the cohesion c_p (0.312 MPa), of the friction angle ϕ_p (33°) and of the elastic modulus E of the rock mass (3772 MPa), through the linearization of the Hoek and Brown criteria, as illustrated by (Hoek, 2006). The residual strength parameters have been made equal to the peak values ($c_r = c_p$; $\phi_r = \phi_p$). The dilatancy angle has been assumed equal to 50% of the friction angle and the Poisson ratio has been made equal to 0.3. The litho static stress p_0 has been taken as 5 MPa. The support structure is made up of shotcrete lining of a thickness of 0.3 m. A mean elastic modulus of the shotcrete E_{con} of 12000 MPa has been hypothesized, as well as a Poisson ratio ν_{con} of 0.15. The stiffness k_{sup} has been calculated equal to 190.2 MPa/m (Oreste, 2003a; Hoek and Brown, 1980). The

distance from the excavation face d at which the support is positioned in the tunnel is equal to 1.5 m.

Given the uncertainty concerning many of the above reported parameters (c_p , ϕ_p , E , p_0 , k_{sup}), it was decided to assume a normal probabilistic distribution for each one of them with a mean value equal to the estimated value and the standard deviation obtained on the basis of the probable variability interval (semi-amplitude of the interval equal to 5% of the estimated value, reliability of the estimation equal to 99.73%). According to this hypothesis, the standard deviation of the distribution is equal to a third of the semi-amplitude of the assumed variability interval. The normal distributions were interrupted at a distance of $\pm 3\sigma$ from the mean value.

The probabilistic parameters were then extracted randomly, according to the Monte-Carlo procedure, in order to obtain a succession of random vectors of dimension 5, in which each component is made up of a random value of one of the five parameters considered variable. The set of the other deterministic parameters, which were instead considered fixed, was then associated to each vector. A load value σ_{Req} , obtained from the calculation according to the modality illustrated in section 2 (through the intersection of the convergence-confinement curve with the intersection line of the support), was then associated to each random vector. The random vector generation procedure comes to an end when the sample of σ_{Req} is considered stable, in relation to the estimation of the mean value and standard deviation of the population (see section 2). In this specific case, 4539 extractions of the uncertain parameters were necessary and the generated sample of σ_{Req} therefore presents the same number of values. The cumulative distribution of the values of σ_{Req} that were obtained are reported in Fig. 2; values of σ_{Req} varying between about 0.79 MPa and about 0.96 MPa can be observed.

The thus obtained sample of σ_{Req} was then used to proceed with the determination of the maximum bending moment M_{max} and of the associated normal force N induced inside the support structure, through the Einstein and Schwartz (1997) calculation method. Together with the probabilistic variables σ_{Req} , the following parameters were considered uncertain and therefore variable, from the probabilistic point of view: The elastic modulus of the ground E , the elastic modulus of the material constituting the support structure E_{sup} , the area of the section of the support structure A_{sup} , the lateral thrust coefficient K_0 in litho static conditions. A normal type probabilistic distribution was also considered for these parameters and interrupted at $\pm 3\sigma$ from the mean value of the distribution. The mean value of each distribution is represented by the estimation value of the parameter ($E = 3772$ MPa, $E_{sup} = 12000$ MPa, $A_{sup} = 0.3$ m², $K_0 = 0.5$); the standard deviation was considered a third of the semi-amplitude of the interval of variability, which was assumed equal to 5% of the mean value.

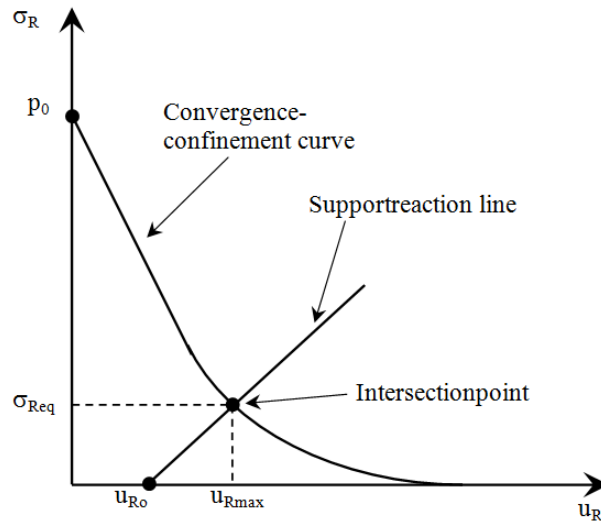


Fig. 1. Intersection between the convergence-confinement curve of the tunnel and the reaction line of the support. The intersection point represents the final equilibrium situation at a long distance from the excavation face (Rechsteiner and Lombardi, 1974; Panet, 1995; Oreste, 2003a; Hoek and Brown, 1980)

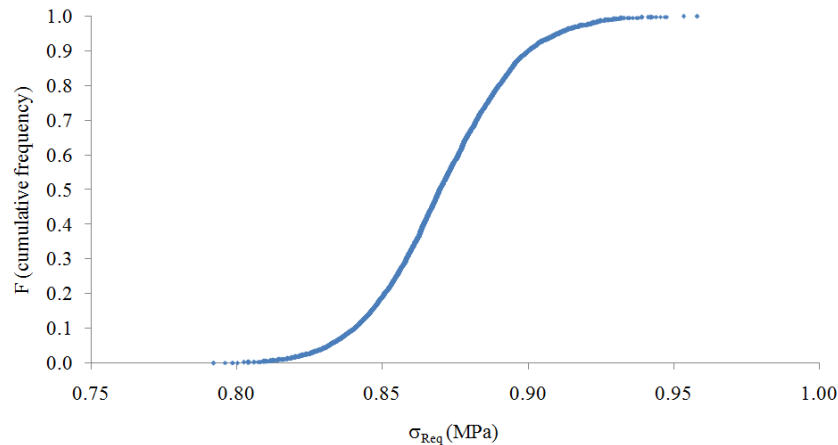


Fig. 2. Cumulative probabilistic distribution of σ_{Req} , obtained from the sample generated using the Monte-Carlo procedure, starting from the normal probabilistic distribution of the parameters considered uncertain

Proceeding with the Monte-Carlo method in the same way as previously seen for the convergence-confinement method for each random vector extracted, which was composed of random values of the five uncertain parameters (σ_{Req} , E , E_{sup} , A_{sup} , K_0), it is possible to calculate the maximum moment M_{max} on the support structure and the associated normal force N . The procedure continues until the available sample of values of σ_{Req} has been completed.

Discussion

The final result is a set of couples of values of M_{max} - N , which can be well represented as a cloud of points in a M_{max} - N diagram (Fig. 3). The cumulative distribution of the maximum moment M_{max} can also be obtained and as a consequence, it is possible to

identify the maximum moment referring to a probability of 99% (0.0168 MN·m/m) (Fig. 4): This value has the probability of being overcome inside the support structure in only 1% of the cases.

It is possible to extract the couples of values M_{max} - N for which M_{max} falls within a relatively restricted interval around the cumulated frequency of 99% from the sample of M_{max} - N couples, for example, between 98.5 and 99.5% (Fig. 3), that is, between 0.01672 and 0.01700 MN·m/M. It is possible to analyze the cumulative distribution of only force N for these couples of values (Fig. 5). By calculating the mean value and the standard deviation of the N of this sub-set of the sample, it is possible to trace the normal cumulative distribution, which is very useful to identify two extreme values of N , for example the force N associated to a cumulative percentage of 5% ($N = 3.813$ MN/m) and a cumulative percentage of 95% ($N = 4.189$ MN/m).

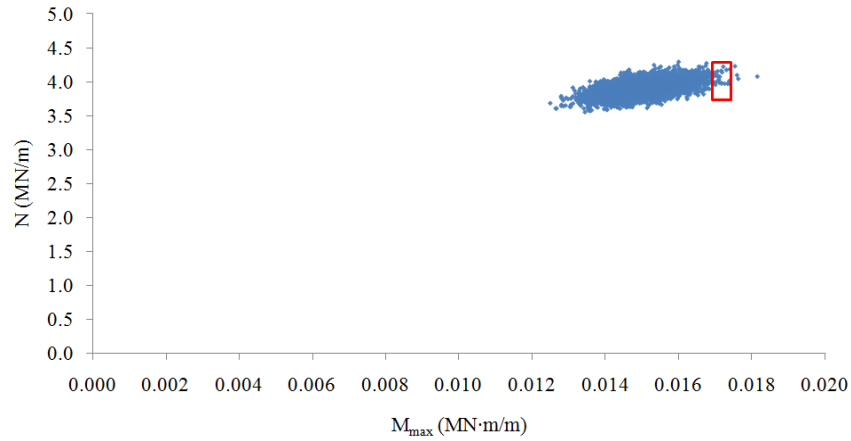


Fig. 3. Results of the Monte-Carlo analyses, in terms of couples of values of the maximum moment M_{max} in the support structure and the associated normal force N . The red rectangle encloses all the points of the sample of the M_{max} - N couples that present a moment M_{max} with a cumulative percentage between 98.5 and 99.5%

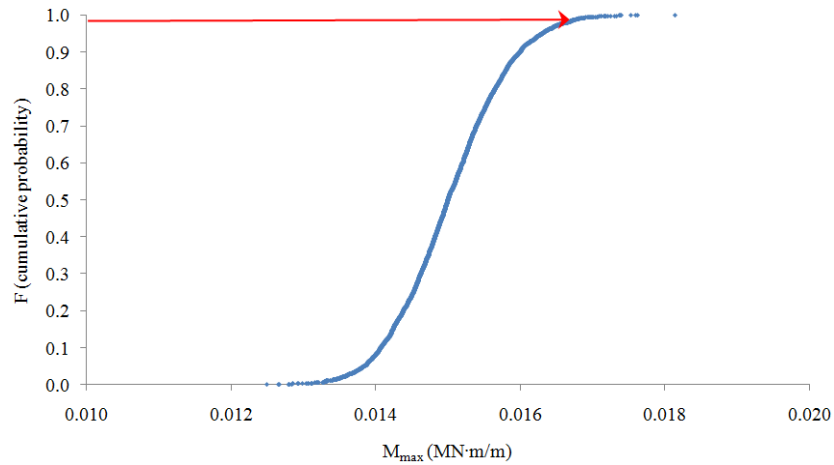


Fig. 4. Cumulative distribution of M_{max} in the support structure, obtained through the Monte-Carlo method. The value of M_{max} identified in the graph shows a cumulative frequency of 99%

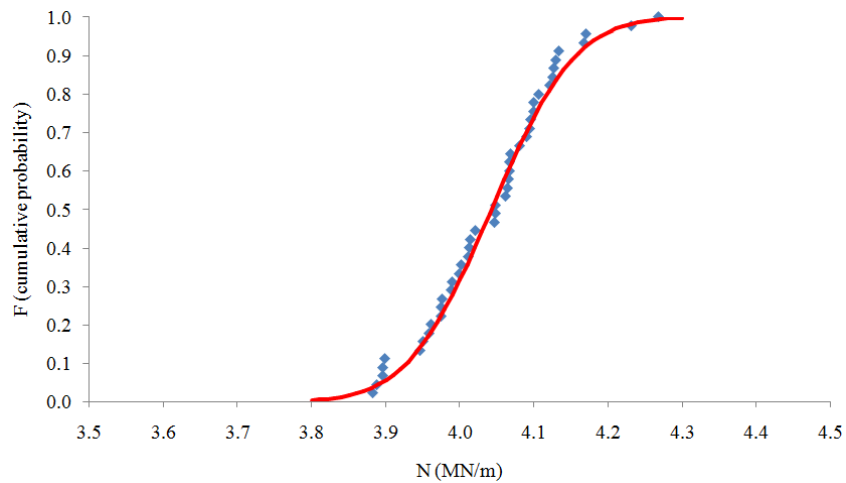


Fig. 5. Cumulative distribution of N for only couples of values of M_{max} - N of the sample that present cumulative frequencies of M_{max} between 98.5 and 99.5%. The cumulated normal distribution, obtained starting from the mean value and the standard deviation of N for only couples of M_{max} - N of the identified sub-set, is shown in red

These two extreme values of N , together with the moment M_{\max} relative to a cumulative percentage of 99% (0.0168 MN·m/m) render it possible to obtain a maximum compression strength and a maximum tensile stress (if it exists) which can be compared with the strength of the material that makes up the support structure (which, in this case, is shotcrete).

Conclusion

The parameters that characterize the ground and support structure of a tunnel are usually only known to a certain extent. In order to be able to analyze the statistic conditions of a tunnel, when there is a support structure, it is useful to consider a probabilistic approach that is able to furnish the probabilistic distribution of the stress actions inside the support structure.

The probabilistic approach generally requires resort to the Monte-Carlo method, which allows the uncertain parameters to be extracted randomly, once the probabilistic distribution has been defined for each of them. Moreover, the probabilistic approach should be associated with an analytical calculation method, with closed-form solution, that can manage the remarkable number of analyses that are necessary in contained times.

A probabilistic type of analysis technique that can be used to analyze the behavior of a tunnel and verify the stability conditions of the support structure is presented in this study. This technique, which makes use of the convergence-confinement method and the Einstein and Schwartz method, allows a sample of couples of values of the maximum moment and the associated normal force to be obtained and these values can then be treated from the probabilistic point of view. In particular, it has been possible to determine the maximum moment of the support structure and two values of the normal force, from which it is possible to obtain the representative values of the maximum compression and tensile stresses. These values can then be compared with the strength of the material that constitutes the support structure.

A calculation example, referring to a real tunnel, has been given. This example has made it possible to follow the proposed procedure step by step.

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Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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