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A Developed Version of Conjugate Gradient Method With a New Parameter for Dai-Liao Conjugacy Condition With an Application to Solar Panel

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Abstract: Conjugate Gradient (CG) methods are broadly employed in solving bigger-scale unconstrained optimization problems. Two famous methods are the Hestenes-Stiefel (HS) as well as Polak-Ribière-Polyak (PRP) CG methods, which usually work well in practice. However, they cannot satisfy the Global Convergence (GC) property. To retain and enhance the previous practical behavior as well as rectify the latter difficulty, this paper constructs a new CG method based on the Dai-Liao conjugacy condition, the Restart Property (RP), and the Lipschitz constant. It is refined that the suggested method meets the sufficient Descent Condition (DC), and the GC properties with the new RP depend on the Lipschitz Constant (LC). To study the behavior of the method, we compared its performance with that of the useful CG-Descent 6.8 as well as non-negative Dai-Liao methods by applying them to 143 optimization problems that are selected from the CUTEst library. The numerical findings indicate that the newly proposed method surpasses the latter two methods as well as other recently published CG methods in terms of the number of iterations, the number of functions, gradient evaluations as well as the CPU time required to solve the problems. In addition, we present an application of the considered CG methods to the solar panel for dust effect and prevent dust accumulation and optimize module performance.

Keywords: Unconstrained Optimization, Conjugate Gradient Methods, Sufficient Descent Condition

Introduction

The optimization problem given below is taken into consideration:

$$Minf\left(\mathrm{x}\right), xR^{n} \tag{1}$$

In which $f: R^n \to R$ resembles a continuous and differentiable function. Moreover, the subsequent assumption holds significant importance in ensuring the convergence analysis with respect to Conjugate Gradient (CG) methods.

Assumption 1

A. The level set $\Omega = \{x \,|\, f(x) \leq f(x_1)\}$ is bounded, inferring the existence of a positive constant ρ given by $\|x\| \leq \rho, \forall x \in \Omega$, where $\|x\|$ resembles the Euclidean norm.
B. In certain neighborhood Q of Ω, f resembles a differentiable continuous function, in which the gradient is also Lipschitz Continuous (LC). This infers that $\forall x, y \in Q$, a constant $L > 0$ exists given

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that $\|g(x) - g(y)\| \leq L\|x - y\|$, where $g(x) = \nabla f(x)$

Furthermore, based on this assumption, we may infer that a positive constant B exists giving:

$\|g(u)\| \leq B, \ldots \forall u \in N$

The CG methods achieve a stationary point by employing an iterative technique that commences from the initial point,x_1, as outlined below:

$$x_{k+1} = x_k + \alpha_k d_k, k = 1, 2, ..., \tag{2}$$

To acquire the step length α_k, we employ two primary line search techniques: The first one is an exact line search as specified below: Suppose we have $\varphi(\alpha) = f(x_k + \alpha d_k)$, which is a sub-problem that emerges from x_k, aims to determine a steplength in the direction d_k, ensuring $\varphi(\alpha_k) < \varphi(0)$. Suppose the step size is specified that it minimizes the search direction given by:

$$f(x_k + \alpha_k d_k) = \underset{\alpha \geq 0}{min} f(x_k + \alpha d_k)$$

This line search is known as optimal line search or exact line search, while α_k is referred to as an optimal steplength. Moreover, in practical calculations, it is often challenging to find the exact optimal step length, especially when the initial point is distant from the optimal solution or when the dimension is larger (Sun & Yuan, 2006). Hence, employing an inexact line search with a lower computational burden is preferable.

The second category is the inexact line search, with the renowned inexact line search methods including:

A. The Strong Wolfe-Powell (SWP) (Wolfe, 1969; 1971) line search conditions defined by:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k \tag{3}$$

And:

$$\sigma g_k^T d_k \leq g(x_k + \alpha_k d_k)^T d_k \leq -\sigma g_k^T d_k \tag{4}$$

in which $0 < \delta < \frac{1}{2}, \delta < \sigma < 1$ and $g_k = g(x_k) = \nabla f$.

B. The weak Wolfe-Powell (WWP) line search condition (3) and:

$$g(x_k + \alpha_k d_k)^T d_k \geq \sigma g_k^T d_k \tag{5}$$

The CG approach's search direction is given by:

$$d_k = \begin{cases} -g_k, & \text{if } k = 1 \\ -g_k + \beta_k d_{k-1}, & \text{if } k \geq 2 \end{cases} \tag{6}$$

In which β_k is called the CG formula.

The famous traditional CG methods are Hestenses–Stiefel (HS) (Hestenes & Stiefel, 1952), Polak–Ribiere–Polyak (PRP) (Polak & Ribiere, 1969) as well as (Liu and Storey, 1991) (LS):

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}} \quad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{\|g_{k-1}\|^2} \quad \beta_k^{LS} = -\frac{g_k^T y_{k-1}}{d_{k-1}^T g_{k-1}}$$

In which:

$$y_{k-1} = g_k - g_{k-1}$$

Meanwhile, Fletcher–Reeves (FR) (Fletcher, 1964), Fletcher (CD) (Al-Baali & Fletcher, 1986); Dai and Yuan (1999) (DY) are given by:

$$\beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2} \quad \beta_k^{CD} = -\frac{\|g_k\|^2}{d_{k-1}^T g_{k-1}} \quad \beta_k^{DY} = \frac{\|g_k\|^2}{d_{k-1}^T y_{k-1}}$$

Dai and Liao (2001) introduced the CG formulation, written as:

$$\beta_k^{DL} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}} - t\frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}} = \beta_k^{HS} - t\frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}} \tag{7}$$

Where, $t \geq 0$. Nonetheless, β_k^{DL}acquire similar problem as β_k^{PRP} as well as β_k^{HS} that β_k^{DL} is positive generally. Therefore, Dai and Liao (2001) reinstated Eq. (7) by:

$$\beta_k^{DL+} = \max\left\{\beta_k^{HS}, 0\right\} - t\frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}} \tag{8}$$

In Hager and Zhang (2005) proposed the subsequent CG method (CG-Descent) derived from Equation (7):

$$\beta_k^{HZ} = \left\{\beta_k^N, \eta_k\right\}$$

in which:

$$\beta_k^N = \frac{1}{d_k^T y_k}\left(y_k - 2d_k\frac{\|y_k\|^2}{d_k^T y_k}\right)^T g_k, \eta_k = -\frac{1}{\|d_k\|\{\eta, \|g_k\|\}}$$

And $\eta > 0$ is a constant.

Remark: Suppose $t = 2\frac{\|y_k\|^2}{s_k^T y_k}$, then $\beta_k^N = \beta_k^{DY}$.

Note that the latest version of CG-Descent is called CG-Descent 6.8 (Hager & Zhang, 2013).

According to Eq. (7), Andrei (2013a-b) introduced a pair of Three-Term CG (TTCG) methods in the year 2013, as outlined below:

$$d_k = -g_k + \eta_k d_{k-1} - \theta_k y_{k-1}, t = \left(1 + \frac{\|y_{k-1}\|^2}{s_{k-1}^T y_{k-1}}\right)$$

$$d_k = -g_k + \eta_k d_{k-1} - \theta_k y_{k-1}, t = \left(1 + \frac{\|y_{k-1}\|^2}{s_{k-1}^T y_{k-1}}\right)$$

In which:

$$\eta_k = \frac{g_k^T y_{k-1} - t g_k^T s_{k-1}}{y_{k-1}^T d_{k-1}}, \theta_k = \frac{g_k^T d_{k-1}}{y_{k-1}^T d_{k-1}}$$

Another alteration derived from Eq. (8), as outlined by Babaie-Kafaki and Ghanbari (2014) is as given below:

$$d_k = -g_k + \eta_k d_{k-1} - \theta_k y_{k-1}, t = \left(\max\left(\varsigma, 1 - \frac{\|y_{k-1}\|^2}{s_{k-1}^T y_{k-1}}\right)\right); \varsigma > 0$$

Meanwhile, Alhawarat *et al.* (2021a) developed the CG formula given below with restart criteria depending on the Lipschitz constant as follows:

$$\beta_k^{AZPRP} = \begin{cases} \frac{\|g_k\|^2 - m_k\left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}, & \|g_k\|^2 > m_k\left|g_k^T g_{k-1}\right| \\ 0, & otherwise \end{cases} \tag{9}$$

Where:

$$m_k = \frac{\|s_k\|}{\|y_k\|}$$

In addition, Alhawarat *et al.* (2021b); and Alhawarat (2023) introduced the CG method defined as:

$$d_k^{FTCGPRP} = -g_k + \left(\beta_k^{HS} - t_k \frac{g_k^T s_{k-1}}{y_{k-1}^T d_{k-1}}\right) d_{k-1} - \left(\frac{g_k^T d_{k-1}}{y_{k-1}^T d_{k-1}}\right)(y_{k-1} + s_{k-1}) \quad (10)$$

Moreover, Alhawarat (2023) presented a preprint for Dai and Liao parameters as follows:

$$\beta_k^{AZHS} = \begin{cases} \frac{\|g_k\|^2 - \mu_k |g_k^T g_{k-1}|}{d_{k-1}^T y_{k-1}} - \frac{1}{\alpha_k}\mu_k \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}, & if\ \|g_k\|^2 > \mu_k |g_k^T g_{k-1}| \\ -\frac{1}{\alpha_k}\mu_k \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}, & otherwise \end{cases}$$

The CG method is applicable across various domains, including image enhancement, neural networks, engineering mathematical challenges as well as numerous others.

The New Search Direction and its Motivation

We devise a novel CG technique founded on the Dai-Lio conjugacy condition with RP reliant on the Lipschitz constant. This adjusted technique can meet the descent criterion and exhibit GC attributes. Furthermore, Eq. (11) proves to be more effective compared to $\beta_k^{AZPRP}$and other renowned methods like CG-Descent6.8 and DL+ in terms of iteration count, function and gradient evaluations as well as CPU time. In addition, the revised search direction is provided as given below:

$$d_k^{FTCGAZPRP} = -g_k + \left(\beta_k^{AZPRP} - (t+1)\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right) d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right)(y_{k-1}) \quad (11)$$

Here, we choose $t = t_k = \min(1, |1 - m_k|)$ so that $0 < t_k \leq 1$. It is clear that if $L \geq 1$, then:

$$t_k = 1 - m_k \leq 1 - \frac{1}{L}$$

It is worth knowing that if $f(x)$ is bi-Lipschitz, it implies the existence of a constant $L > 1$ given that:

$$L\|x - y\| \leq \|g(x) - g(y)\| \leq L\|x - y\|$$

Then, $t_k = 1 - m_k$ and we obtain $0 < t_k \leq 1$. Moreover, we observed from the numerical findings that the value of $m_k < 1$ for almost all iterations, and rarely its value becomes greater than one. Now, the important question is $m_k \to \infty$? By the definition of $m_k = \frac{\|s_k\|}{\|y_k\|}$, it goes ∞ iff $y_k \to 0$. This means that $x_{k+1} \to x_k$ and we know that $x_{k+1} = x_k + \alpha_k d_k$. Thus, $\alpha_k d_k \to 0$. However, $\alpha_k > 0$- see reference (Dai & Liao, 2001) which implies that $d_k \to 0$. Using Theorem 4.1 and Lemma 4.3 we reach a contradiction.

This study presumes the subsequent Eq. (11):

$$\eta_k = \beta_k^{AZPRP} - t_k \frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}, \theta_k = \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right)$$

Note that Eq. (11) may be simplified to the PRP CG method when employing an exact line search. Note that this is due to the application of exact line search properties, resulting in:

$$g_k^T d_{k-1} = 0$$

Provided that $g_k^T s_{k-1} = \alpha_k g_k^T d_{k-1}$, we now have:

$$d_k^{FTCGPRP} = -g_k + \left(\beta_k^{AZPRP}\right) d_{k-1}$$

Algorithm 1 outlines the procedure of the CG method to acquire stationary points utilizing SWP line search as well as Eq. (11) Alongside the stopping criteria:

$$\|g_k\| \leq 10^{-6}$$

Algorithm

The algorithm is the main part of numerical analysis. As an example, the cod for algorithm 1 with CG-Descent formula and approximation of Wolfe-Powell line search may be gained from the Hager webpage https://people.clas.ufl.edu/hager/software/

The current release is version 6.8. Another website (Gilbert & Nocedal, 1992) provided the code for solving CG+ implementing three distinct CG method versions, known as the FR method, the PRP method as well as the positive PRP method (Beta is always positive). Moreover, the coding has been established at the Optimization Center, a joint venture of Northwestern University as well as Argonne National Laboratory CG+ Non-linear Optimization Code (northwestern.edu). The standard test functions may be taken from the link CUTEr/st Test Problem Set (rl.ac.uk).

Algorithm 1

Step 1. Set the initial point x_1. Here, the initial direction $is d_1 = -g_1$. Set $k: = 1$

Step 2. If the stopping condition is satisfied, the process should be terminated.

Step 3. The search direction d_k is calculated relying on Eq. (2) employing Eq. (11).

Step 4. The step sizes α_k are calculated utilizing Eqns. (3-4).

Step 5. Update x_k depending on Eq. (2).

Step 6. Set $k: = k + 1$ and move to Step 2.

GC analysis of the CG method with Algorithm 1

The Descent Condition (DC) is formulated by $g_k^T d_k < 0, \ldots \forall k \geq 1$, is beneficial in the examination of the CG method, which has a crucial part in the verification of GC analysis. Here, (Al-Baali, 1985) altered the DC to a sufficient DC:

$$g_k^T d_k \le -c\left\|g_k\right\|^2, \ldots \forall k \ge 1 \tag{12}$$

In which $c > 0$. Al-Baali and Latif (2022) suggest an algorithm for tackling non-linear unconstrained optimization issues by merging an elongated CG technique and the dampened method of Al Baali-Powell for the BFGS method.

In the subsequent theorem, we demonstrate that the exploration direction in Eq. (11) fulfills the adequate descent criterion (12).

Lemma 4.1. If $L > 1$, then the following equation holds:

$$\left(\frac{\|g_k\|^2 - m_k \cdot \left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T\left(y_{k-1}\right) \le 0$$

Proof. If $L > 1$then $\mu_k < 1$, thus we have the following cases:

$$g_k^T g_{k-1} < 0 \; and \; g_k^T d_{k-1} > 0$$

Or:

$$g_k^T g_{k-1} > 0 \; and \; g_k^T d_{k-1} < 0$$

Then, the following conditions are satisfied:

$$\left(\frac{\|g_k\|^2 - m_k \cdot \left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T\left(y_{k-1}\right) < 0$$

However, by using Eq. (6) and SWP line search, the following inequalities cannot be satisfied:

$$g_k^T g_{k-1} > 0 \; and \; g_k^T d_{k-1} > 0$$

Or:

$$g_k^T g_{k-1} < 0 \; and \; g_k^T d_{k-1} < 0$$

Theorem 4.1. Assume that the sequences $\{x_k\}$ and $\{d_k\}$ be produced utilizing Eqs. (2-11), with α_k determined by SWP line search. Subsequently, the condition of sufficient DC (12) is met.

Proof. Multiplying Eq. (11) by g_k^T yields:

$$g_k^T d_k = -\|g_k\|^2 + \left(\frac{\|g_k\|^2 - m_k \cdot \left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2} - t\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T\left(y_{k-1} + d_{k-1}\right)$$

$$= -\|g_k\|^2 + \left(\frac{\|g_k\|^2 - m_k \cdot \left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - t\left(\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T y_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1}$$

$$= -\|g_k\|^2 - t\alpha_k\left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} + \left(\frac{\|g_k\|^2 - m_k \cdot \left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T y_{k-1}$$

$$= -\|g_k\|^2 - (t+1)\left(\frac{\left\|g_k^T d_{k-1}\right\|^2}{\|g_{k-1}\|^2}\right). + \left(\frac{\|g_k\|^2 - m_k \cdot \left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}\right) g_k^T d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right) g_k^T y_{k-1}$$

By utilizing Lemma 4.1, we gain the following result:

$$g_k^T d_k \le -\|g_k\|^2$$

The completion of the proof is obtained.

Lemma 4.2. If $\|g_k\|^2 > \mu_k\left|g_k^T g_{k-1}\right|$ and $L > 1$, then:

$$\|g_k\|^2 - \frac{1}{L}\left|g_k^T g_{k-1}\right| \le L\left|\|g_k\|^2 - \left|g_k^T g_{k-1}\right|\right| \tag{13}$$

Proof. We will carry out the proof utilizing contradiction. Let's assume that:

$$\|g_k\|^2 - \frac{1}{L}\left|g_k^T g_{k-1}\right| > L\left|\|g_k\|^2 - \left|g_k^T g_{k-1}\right|\right|$$

We now divide both sides by L yielding:

$$\frac{\|g_k\|^2}{L} - \frac{1}{L^2}\left|g_k^T g_{k-1}\right| > \left|\|g_k\|^2 - \left|g_k^T g_{k-1}\right|\right| \tag{14}$$

Since $L > 1$. We deduce that the inequality formulated in (14) is false, leading to a contradiction. Consequently, inequality (13) holds true.

The Zoutendijk (1970) condition introduced a helpful Lemma for examining the GC characteristic of the CG method. Here, the Lemma is provided as follows:

Lemma 4.3. We assume that Assumption 1 remains valid. Take into account that any CG method in the form of (2) holds the DC and α_k adheres to the WWP line search (3) and (5), where the search direction descends. Consequently, the subsequent condition is upheld:

$$\sum_{k=0}^{\infty} \frac{\left(g_k^T d_k\right)^2}{\|d_k\|^2} < \infty \tag{15}$$

Dai and Liao (2001) provide the following corollary

Corollary 4.1. Suppose that Assumption 1 holds. Take into account that any CG method in the form (2) satisfies the DC and α_k adheres to the WWP line search (3) and (5), in which the search direction descends. If:

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^t}{\|d_k\|^2} = \infty$$

For any $t \in [0, 4]$, then the method converges.

Dai and Liao (2001) provide a helpful theorem for acquiring the GC theorem of the CG method as given below:

Theorem 4.2. Assuming Assumption 1 is valid. Take into account any CG method presented in Eq. (2), in which d_k satisfies sufficient descent direction while α_k is derived from the robust Wolfe line search. Provided that:

$$\sum_{k \ge 1}^{\infty} \frac{1}{\|d_k\|^2} = \infty$$

Hence:

$\lim_{k\to\infty}\inf\|g_k\| = 0$

GC of Algorithm 1 Having General Non-Linear Functions

Establishing the convergence analysis for our novel search direction relies heavily on the subsequent limitations for η_k. The primary significance of this limitation is to prevent the multiplayer of the CG method from being positive:

$$\|\xi\|\,\|d_k\| = \left\|-g_k + \beta_{k-1}^{(2)} d_{k-1}\right\| \le \|g_k\| + \left\|\beta_{k-1}^{(2)}\right\|\,\|d_{k-1}\| \tag{18}$$

$$\eta_k^+ = \max\left\{0, \beta_k^{AZPRP} - t\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right\}$$

Therefore, Eq. (12) will subsequently become:

$$d_k^{FTCGAZPRP} = -g_k + \left(\beta_k^{AZPRP} - t_k\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right) d_{k-1} - \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right)(y_{k-1} + d_{k-1}),$$

$$d_k = -g_k + \eta_k^+ d_{k-1} - \theta_k (y_{k-1} + d_{k-1})$$

Where:

$$\theta_k = \left(\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}\right)$$

The subsequent lemmas correspond to Lemma 4.1 as well as Lemma 4.2, as outlined by Gilbert and Nocedal (1992).

Lemma 4.4. Take into consideration that Assumption 1 is valid. Moreover, the sequences $\{g_k\}$, as well as $\{d_k\}$, are produced utilizing Algorithm 1, in which the step size α_k is calculated through the SWP line search, ensuring that sufficient DC holds. Suppose we have $\beta_k \ge 0$. Therefore, there exists a constant $\gamma > 0$, such that $0 < \gamma \le \|g_k\| \le \overline{\gamma}$ for all $k \ge 1$. Hence, $d_k \vDash 0$ while:

$\sum_{k=0}^{\infty} \|u_{k+1} - u_k\|^2 < \infty$, in which $u_k = \frac{d_k}{\|d_k\|}$.

Proof. First, provided that $d_k = 0$. Therefore, the sufficient DC yields $g_k = 0$. Hence, we assume that $d_k \vDash 0$ while:

$$\|g_k\| \ge \gamma, where\gamma > 0 \tag{16}$$

We partition Eq. (11) into two segments, as described below:

$$\beta_k^{(1)} = \frac{\|g_k\|^2 - \mu_k\left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}$$

$$\beta_k^{(2)} = -t\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}$$

We then define the following:

$$\xi = \frac{\left\|-g_k + \beta_k^{(2)} d_{k-1}\right\|}{\|d_k\|}, \zeta = \frac{\beta_k^{(1)}\|d_{k-1}\|}{\|d_k\|}$$

The definition of u_k yields:

$$u_k = \frac{d_k}{\|d_k\|} = \frac{-g_k + \left(\beta_k^{(1)} + \beta_k^{(2)}\right) d_{k-1}}{\|d_k\|} = \xi + \zeta\frac{d_{k-1}}{\|d_k\|} = \xi + \zeta u_{k-1}$$

Because the UK is a unit vector, we then have:

$$\|\xi\| = \|u_k - \zeta u_{k-1}\| = \|\zeta u_k - u_{k-1}\|$$

Utilizing the triangle inequality and $\zeta > 0$ yields:

$$\|u_k - u_{k-1}\| = 2\,\|\xi\| \tag{17}$$

Subsequently, utilizing definition ξ yields:

$$\left|\beta_k^{(2)}\right| = \left|-t\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right| \le t\frac{\|g_k\|\|s_{k-1}\|}{\|g_{k-1}\|^2}$$

By utilizing Eq. (18), we gain the inequalities below:

$$\|\xi\|\,\|d_k\| \le \|g_k\| + \frac{t}{\alpha_k}\frac{\|g_k\|\|s_{k-1}\|^2}{\|g_{k-1}\|^2} \le \gamma + \frac{t\gamma B^2}{\alpha_k\overline{\gamma}^2}$$

Next, utilizing Eq. (17) yields:

$$\|u_k - u_{k-1}\| = 2\,\|\xi\| = 2\frac{\gamma + \frac{t\gamma B^2}{\alpha_k\overline{\gamma}^2}}{\|d_k\|}$$

Which completes the proof.

Gilbert and Nocedal (1992) introduced Property* in their paper.

*Property**

Set any CG method expressed in the form of Eqns. (1-2). Let:

$$0 < \gamma \le \|g_k\| \le \overline{\gamma} \tag{19}$$

For all $k \ge 1$. The CG method then acquires Property (*) provided that $\forall k$, constants $b > 1$ and $\lambda > 0$ exist, such that $|\beta_k| \le b$ and $\|s_k\| \le \lambda$, which yields $|\beta_k| \le \frac{1}{2b}$.

Lemma 4.5. We incorporate the CG method outlined in Eqns. (2) as well as (6), incorporating the parameter η_k^+, in which the step size adheres to SWP line search (3) as well as (4). Provided that condition (19) is met. Then, η_k^+ exhibits Property* provided that Eq. (16) is true. Therefore, $\exists b > 1$ as well as $\lambda > 0$ such that $\forall k \ge 1$, we obtain $\left|\eta_k^+\right| \le b$ while if $\|s_k\| \le \lambda$, we get $\left|\eta_k^+\right| \le \frac{1}{2b}$.

Proof. Here, we set $b = \frac{2(L+t)\overline{\gamma}B}{\gamma^2} \ge 1$, while $\lambda = \frac{\gamma^2}{2b(L+t)\overline{\gamma}}$.

Utilizing SWP (3) as well as (4) with Eq. (19) yields:

$$\left|\eta_k^+\right| \le \left|\frac{\|g_k\|^2 - m_k.\left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}\right| + t\left|\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right| \le \left|\frac{\|g_k\|^2 + .\left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|^2}\right| + t\left|\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right| \le \frac{\|g_k\|(\|g_k\| + \|g_{k-1}\| + t\|s_{k-1}\|)}{\gamma^2} \le \frac{\left(2\overline{\gamma}^2 + tB\right)}{\gamma^2} = b > 1$$

if $\|s_k\| \le \lambda$.

To show that $\beta_k^{AZPRP} \le 1/2b$, we divide the proof into two subsequent cases.

Case A: $\mu_k < 1$. Provided that $\mu_k < 1$. Here, we conclude that $L > 1$ by using Lemma 4.2:

$$\beta_k^{AZPRP} \le \frac{(L)\|g_k\|\|g_k - g_{k-1}\|}{\|g_{k-1}\|^2}$$

$$|\eta_k^+| \le \frac{(L)\|g_k\|\|g_k - g_{k-1}\|}{\|g_{k-1}\|^2} + t\left|\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right| \le \frac{L(L)\|g_k\|\|s_{k-1}\| + t\|g_k\|\|s_{k-1}\|}{\gamma^2} \le \frac{(L(L)+t)\overline{\gamma}\lambda}{\gamma^2}$$

$$|\eta_k^+| \le \frac{1}{2b}$$

Case B: $\mu_k \ge 1$. Utilizing Assumption 1 and Eq. (12) yields:

$$|\beta_k^{AZPRP}| \le \frac{\|g_k\|^2 - \mu_k |g_k^T g_{k-1}|}{\|g_{k-1}\|^2} \le \frac{\|g_k\|^2 - |g_k^T g_{k-1}|}{\|g_{k-1}\|^2}$$

$$\le \frac{\|g_k\|\|g_k - g_{k-1}\|}{\|g_{k-1}\|^2} \le \frac{L\lambda\overline{\gamma}}{\gamma^2}$$

$$|\eta_k^+| \le \frac{L\|g_k\|\|g_k - g_{k-1}\|}{\|g_{k-1}\|^2} + t\left|\frac{g_k^T s_{k-1}}{\|g_{k-1}\|^2}\right| \le \frac{L^2\|g_k\|\|s_{k-1}\| + t\|g_k\|\|s_{k-1}\|}{\gamma^2} \le \frac{(L^2+t)\overline{\gamma}\lambda}{\gamma^2}$$

$$\lambda = \frac{\gamma^2}{(L+t)2b}.$$

The completion of the proof is obtained.

The Lemma, as well as the theorem below, bear a resemblance to those introduced by Gilbert and Nocedal (1992). In this instance, Lemma 4.4 is presented without its proof, which is available for reference in Gilbert and Nocedal (1992).

Lemma 4.6. Suppose Assumption 1 remains valid. Additionally, suppose sequences $\{g_k\}$, as well as $\{d_k\}$, are produced by Algorithm 1, where α_k is calculated through the WWP line search. Under these circumstances, the sufficient DC is satisfied, provided that the method possesses Property*. Provided also that $\|g_k\| \ge \gamma$ for several $\lambda > 0$. Therefore, $\exists \lambda > 0$ such that for arbitrary $\Delta \in \mathbb{N}$as well as arbitrary index k_0, an index $k > k_0$ exists satisfying:

$$|\kappa_{k,\Delta}^{\lambda}| > \frac{\lambda}{2}$$

In which $\kappa_{k,\Delta}^{\lambda} = \{i \in \mathbb{N}: k \le i \le k + \Delta - 1, \|s_i\| > \lambda\}$, $\mathbb{N}$ resembles the positive integers set, while $|\kappa_{k,\Delta}^{\lambda}|$ expresses the number of elements in $\kappa_{k,\Delta}^{\lambda}$.

Theorem 4.2. Let Assumption 1 remain valid. In addition, note that the sequences $\{g_k\}$, as well as $\{d_k\}$, are formulated via Algorithm 1, wherein α_k is measured using the WWP line search method and satisfies the sufficient DC. Furthermore, let Property* be upheld. Therefore, we obtain $lim_{k\to\infty} \inf \|g_k\| = 0$.

Results and Discussion

To examine the efficacy of the novel search direction given in Eq. (11), we opted for more than 140 test functions from CUTEst in Bongartz *et al.* (1995) with dimension $2 \le n \le 12320$. A comparison with other well-known and robust CG coefficients was carried out, which included CG-Descent6.8, DL+ CG formula as well as FTCGAZPRP based on CPU time, function evaluation count, iteration count, and gradient evaluation count. We employed the SWP line search method to determine the step length using $\delta = 0.01$ as well as $\sigma = 0.1$ for all algorithms except CG-Descent, for which we used an approximate WWP line search as specified by the researchers. Here, the findings of DL+ CG as well as FTCGAZPRP methods were gained by performing the altered code of CG-Descent.

The gradient norm, precisely $\|g_k\| \le 10^{-6}$, was utilized as the stopping condition for all techniques. The computer used is an AMD A4-7210 APU Radeon R3 Graphics with 4 GB of RAM, running Ubuntu 20.04.2.0 LTS. Moreover, the outcomes are depicted in Figs. (1-4), employing a performance metric established by Dolan and Moré (2002).

Figure (1) infers that FTCGAZPRP substantially surpasses the CG-Descent6.8 as well as the DL+ CG formula in terms of the number of iterations. Apart from that, Figure (2) depicts the number of function evaluations, indicating that the FTCGAZPRP** method also markedly surpasses the CG-Descent as well as DL+ methods. In Figs. (3 as well as 4), we observe that the FTCGAZPRP method marginally outperformed CG-Descent and DL+ as we utilize SWP line search. Nonetheless, if the SWP line search is extended, we anticipate that the efficiency of gradient evaluation will improve for DL+ and FTCGAZPRP.

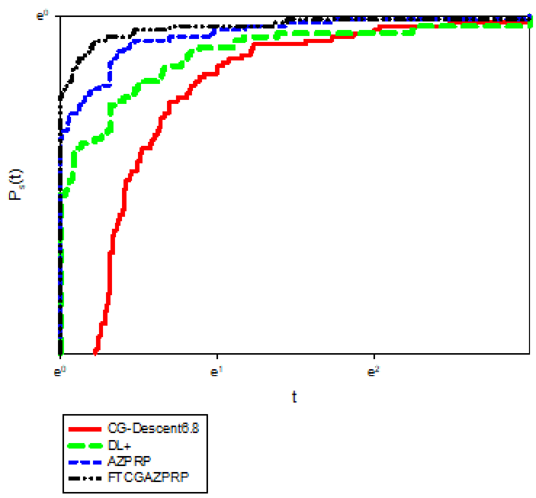

Fig. 1: Measure of performance relying on the number of iterations

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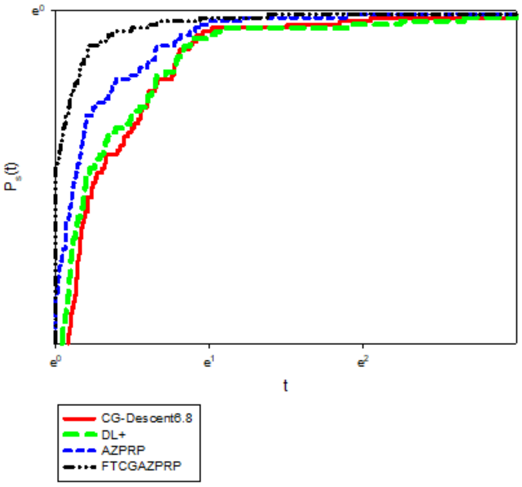

Fig. 2: Measure of performance relying on the function evaluation

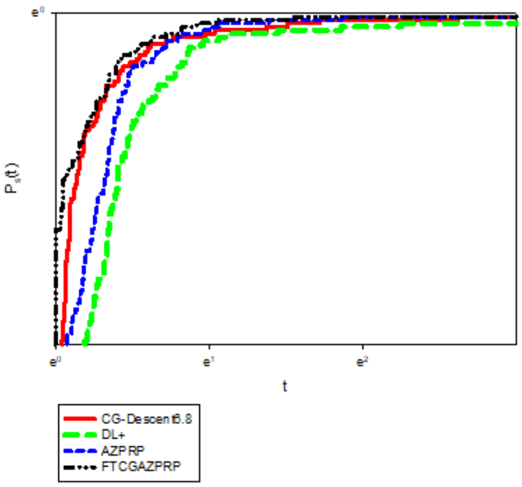

Fig. 3: Measure of performance relying on the gradient evaluation

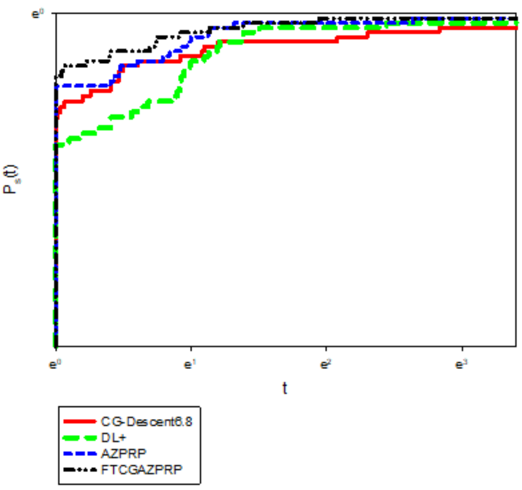

Fig. 4: Measure of performance relying on the CPU time

Application for Solar Panel

Despite fossil fuels currently meeting most of the industrial energy demands, their limited and dwindling resources, coupled with increasing environmental concerns, for instance, climate change as well as global warming (Hermanson *et al.*, 2022; USGCRP, 2018), pose significant risks to mankind.

Consequently, there is an escalating need to increasingly depend on renewable energy resources like hydro, wind, solar, biomass as well as geothermal energies. Utilizing PV solar modules has experienced a notable rise in the last decade. These modules have effectively shifted from large-scale lab models to smaller applications, resulting in a rise in commercial production as well as sales of PVs. Presently, PVs are not only widespread in households or off-grid settings but are also being deployed in large-scale power plants, supplying electricity not only to districts but also to entire cities globally. However, despite the benefits of clean energy from solar cells, this field faces several challenges that hinder conversion efficiency, including environmental variables, for instance, temperature (Schwingshackl *et al.*, 2013), humidity, rainfall(Simsek *et al.*, 2021), seasonal variations, dust and wind speed (Bhattacharya *et al.*, 2014) as well as direction. In conjunction with environmental factors, installation variables like positioning, seasonal alterations in incline, angle of inclination (Hailu & Fung, 2019), elevation as well as installation location also affect the performance of solar panels (Gupta *et al.*, 2019). The dust accumulation on solar panels significantly diminishes incoming solar radiation to photovoltaic modules. Over recent decades, researchers have extensively investigated as well as explained the efficiency losses caused by clouds and dust.

Elminir *et al.* (2006) showed that the decrement in glass transmittance due to dust is impacted by attributes like deposition density, plate tilt angle, as well as surface orientation relative to wind direction. Kaldellis and Kokala (2010) studied dust effects in Athens. They found that a small dust deposition density (around 1 g/m²) can significantly impact PV-panel performance, leading to energy.

production reductions of up to approximately 6.5%. This reduction translates to an annual income loss of approximately €40/kWp, representing 1% of the present turnkey-specific price with respect to domestic PV generators.

In areas without rain for extended periods, the daily energy loss is, in general, much larger. Experimental investigations by Saidan *et al.* (2016) inferred that dust accumulation on photovoltaic solar modules in a desert environment caused average performance degradation rates of 6.24%, 11.8%, as well as 18.74% for exposure periods of 1 day, 1 week, and 1 month, accordingly. Al-

Sudany (2009) studied natural dust deposition on solar panels in Baghdad and found that transmittance decreased by approximately 50% over one month, highlighting the significant impact of the accumulation period on performance degradation.

Said and Walwil (2014) reported a 35% reduction in spectral transmittance due to dust on PV module glass covers. Kazem *et al.* (2014) and Darwish *et al.* (2015) confirmed performance degradation in multicrystalline PV modules due to various pollutants. Mekhilef *et al.* (2012) established a correlation between dust thickness on PV modules and efficiency reduction, noting a significant output drop of 10-20% with heavy dust layers. In contrast, small amounts of dust had minimal effect on sunlight transmission.

The type of glass covering the panel surface, size of dust particles, solar radiation intensity, dust quantity, and dust weight all impact the power produced by a PV panel (Darwish *et al.*, 2013). Furthermore, a variety of pollutants, including ash, sand, soil, and silica, can accumulate on PV panels, affecting their electrical performance. These particles have distinct chemical, physical, as well as structural characteristics relying on environmental factors like air quality, temperature, and humidity. A number of research studies have explored the consequences of dust settling on PV panels, consistently revealing a decrement in efficiency.

Karmouch and Hor (2017) carried out practical experiments in the Jazan region (KSA) to assess the effects of dust accumulation on commercial solar cells at various inclinations. Findings indicate that the settling of dust from the air onto solar panels significantly reduces the short circuit current over a 15-week period for both inclinations, consequently diminishing the solar cells' efficiency. This decline is attributed to reduced transmittance, influenced by dust deposition density as well as particle sizes, which are, in turn, affected by exposure duration and tilt angle. Efficiency reduction amounts to around 10.4% after 16 weeks for panels angled at 30° as well as 9.7% for panels angled at 55°, even with relatively short exposure to dust (Figure 5). Limited rainfall exacerbates the situation, necessitating natural water cleaning to restore efficiency, albeit time-consuming for large solar panel arrays. A self-cleaning system is strongly recommended for dusty regions like Jazan to prevent dust accumulation and optimize module performance.

Figure (5) infers that the characteristics of the solar cell panel measured for 16 weeks tilted at 30°. The relationship between *x* and *y* is a parabola relation. Hence, the regression function may be inferred by:

$$r = w_2x^2 + w_1x + w_0 \quad (20)$$

where, w_0, w_1 and $w_2$are the regression parameters. By using the least square method, we want to solve:

$$\min Q = \sum_{j=1}^{n} \left(y_j - \left(w_0 + w_1x_j + w_2x_j^2\right)\right)^2$$

This equation can be converted into an unconstrained optimization problem in the following manner:

$$\min_{w \in R^3} \sum_{j=1}^{n} f(w) = \sum_{j=1}^{n} \left(y_j - \left(w_0 + w_1x_j + w_2x_j^2\right)\right)$$

By using Algorithm 1, we can obtain the following results:

$w_2 = 0.0043, w_1 =$ -0.1063$, w_0 = 7.8745$

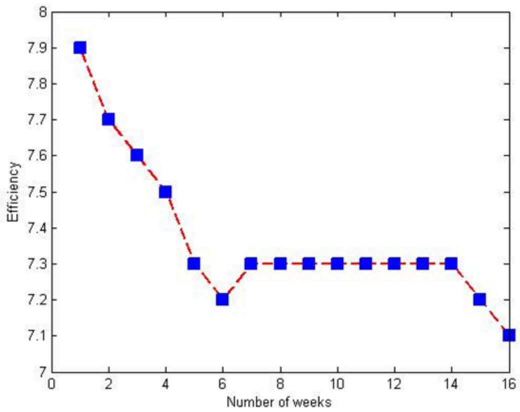

Fig. 5: Characteristics of the solar cell panel measured for 16 weeks

Application of Using the CG Method in a Regression Problem

The following table (Table 1) represents the rate of people who have medical checkups for different clinics.

The relationship that exists between *x* as well as *y* is a parabola relation. Thus, the regression function may be expressed by:

$$r = w_2x^2 + w_1x + w_0 \quad (21)$$

where, w_0, w_1, w_2, are the regression parameters. Utilizing the Least Square Method (LSM) yields:

$$\min Q = \sum_{j=1}^{n} \left(y_j - \left(w_0 + w_1x_j + w_2x_j^2\right)\right)^2$$

This equation may be transformed into an unconstrained optimization problem given by:

$$\min_{w \in R^3} \sum_{j=1}^{n} f(w) = \sum_{j=1}^{n} \left(y_j - \psi\left(1 + x_j + x_j^2\right)^T\right)^2$$

Using the first five data points from Table (1) and applying the poly fit function in MATLAB, we can derive the following values.:

$w_2 = -0.0814 \times 10^5, w_1 = 0.9153 \times 10^5, w_0 = 7.2543 \times 10^5$

By using the extreme value of calculus, we obtain the following solution:

$w_2 = -8138, w_1 = \frac{457651}{5}, w_0 = \frac{3627127}{5}$

By using Algorithm 1, we obtain the following results:

$w_2 = -0.0814 \times 10^5, w_1 = 0.9153 \times 10^5, w_0 = 7.2543 \times 10^5$

Thus, if we substitute the value w_2, w_1, w_0 using Algorithm 1 in Eq. (21) and setting $x = 6$, we found that the approximate solution is given by:

$r = 9.8173\text{e}+005$

Where the relative error is 0.0113.

Table 1: The number of people who review the medical clinics

Number of data (x)	Year	Number of people who have medical checkups (y)
1	2010	796693
2	2011	903233
3	2012	917624
4	2013	946239
5	2014	988701
6	2015	970,785

Conclusion

This study introduces an enhanced version of a novel CG method, which is founded on the Dai-Lio conjugacy condition with an RP dependent on the Lipschitz constant. The adapted method meets the DC and exhibits GC properties. Additionally, numerical outputs demonstrate that the novel approach surpasses the most renowned CG methods of this era in terms of function evaluations, CPU time, gradient evaluations as well as iteration count. Finally, we illustrate the CG method application in assessing the impact of dust on solar panels to prevent dust accumulation and optimize module performance.

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Author's Contributions

Ahmad Alhawarat and Shahrina Ismail: Contributed to the manuscript by writing the modifications.

Ramadan Sabra: Contributed by writing the mathematical proofs.

Mehiddin Al-Baali: Reviewed and provided feedback on the entire manuscript.

Hamid El Hor: Modified and refined the code implementation.

Ali Jaradat: Developed and implemented the application.

The authors made equal and substantial contributions to the composition of this study. All authors have reviewed and endorsed the final manuscript.

Ethics

This manuscript is an original work. The corresponding author certifies that all co-authors have reviewed and approved the final version of the manuscript. No ethical concerns are associated with this submission.

Availability of Data and Material

The data is accessible within the paper.

Competing Interests

The authors state that they do not have any conflicting interests.

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